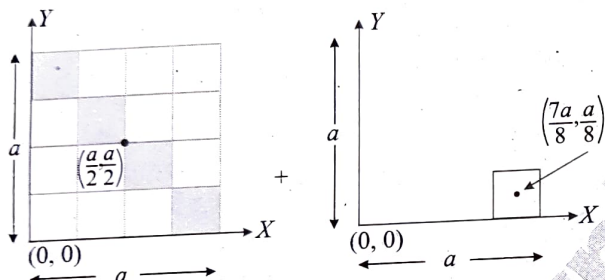
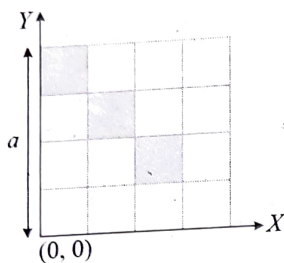


CHAPTER 1

DPP 1.1

Subjective Type

1. Suppose mass of each small square portion is M . This can be expressed as a combination of the two mass systems as given below



Therefore centre of mass of the plate is given by

$$\bar{R}_{cm} = \frac{m_1 \bar{r}_1 + m_2 \bar{r}_2}{m_1 + m_2}$$

$$= \frac{12 \left(\frac{a}{2} \hat{i} + \frac{a}{2} \hat{j} \right) + 1 \left(\frac{7a}{8} \hat{i} + \frac{a}{8} \hat{j} \right)}{13} = \left(\frac{55a}{104} \hat{i} + \frac{49a}{104} \hat{j} \right)$$

Coordinates of centre of mass are $\left(\frac{55a}{104}, \frac{49a}{104} \right)$

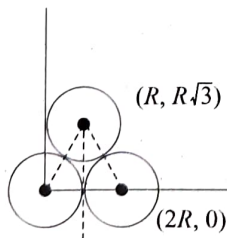
Single Correct Answer Type

2. (b) If mass is non-uniformly distributed, then centre of mass of ring may lie from origin to circumference. Hence $0 \leq b \leq a$.

3. (a) x and y coordinates of all the particles are positive. Hence x and y coordinates of centre of mass of the system have to be positive.

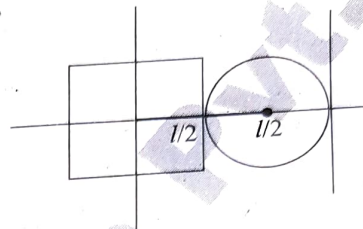
4. (d) $x_{cm} = \frac{m \times 0 + m \times 2R + mR}{3m} = R$

$y_{cm} = \frac{m \times 0 + m \times 0 + mR\sqrt{3}}{3m} = \frac{R}{\sqrt{3}}$



5. (c) $y_{cm} = \frac{m \cdot 0 + m \cdot 0 + m \cdot b + mb + m^2 b}{5m} = \frac{4b}{5}$

6. (a) Mass of square plate $M_1 = l^2 \sigma$
Mass of circular disc $M_2 = \pi \frac{l^2}{4}$



$$x_{cm} = \frac{M_1 \times 0 + \frac{\pi l^2 \sigma l}{4}}{l^2 \sigma \left[1 + \frac{\pi}{4} \right]} \Rightarrow x_{cm} = \frac{\pi l}{4(\pi + 4)} = \frac{\pi l}{\pi + 4}$$

$x_{cm} < \frac{l}{2}$, i.e., COM lies in the square

7. (d) Unless $m_1 = m_3$ the COM of all the four particles can never be at the centre of the square.

8. (a) $X_{cm} = \frac{m \times 0 + ml + m \times 2l + \frac{m}{2} \frac{3l}{2}}{3.5m}$

$$X_{cm} = \frac{15ml}{4 \times \frac{7}{2}m} = \frac{15l}{14}$$

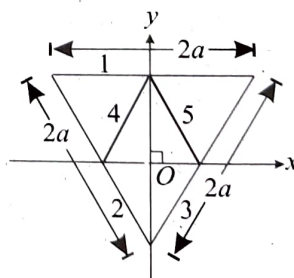
$$Y_{cm} = \frac{m \times 0 + ml + ml + \frac{m}{2} \frac{2l}{2}}{\frac{7}{2}m} \Rightarrow Y_{cm} = \frac{3ml}{7m} = \frac{6l}{7}$$

9. (b) The y -coordinate of centre of mass is

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 + m_5 y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

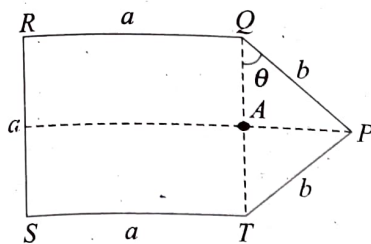
$$= \frac{2m \left(\frac{\sqrt{3}}{2} a \right) + 2m(0) + 2m(0) + m \left(\frac{\sqrt{3}}{4} a \right) + m \left(\frac{\sqrt{3}}{4} a \right)}{8m}$$

$$= \frac{9a}{16\sqrt{3}}$$



10. (d) If centre of mass is at A

$$a^2 \sigma \frac{a}{2} = \sigma \frac{1}{2} ab \sin \theta \frac{1}{3} b \sin \theta$$



$$\text{or } \frac{b}{a} = \sqrt{\frac{13}{4}}$$

$$(c) m_1 x_1 = m_2 x_2$$

$$\Rightarrow m \frac{\sqrt{3}}{2} l = 7m x_2 \Rightarrow x_2 = \frac{\sqrt{3}l}{14}$$

(b) Hence position of collision from O = 2 m.

(c) Mass of disc

If mass of removed disc is m the mass of big disc will be 16 m.

$$x_{cm} = \frac{16m(0) - m(0) - m(3R)}{16m - m - m} = -\frac{3}{14}R$$

Because of symmetry we can say y_{cm} will also have the same magnitude

$$y_{cm} = -\frac{3}{14}R$$

$$\vec{r}_{cm} = -\frac{3}{14}R(\hat{i} + \hat{j})$$

14. (d) Centre of mass of circular disc of radius $4R = (0, 0)$

Centre of mass of upper disc = $(0, 3R)$

Centre of mass of lower disc = $(3R, 0)$

Let M be mass of complete disc and then the mass of cut out disc are

Hence, centre of mass of new structure is given by

$$\bar{x} = \frac{m_1 x_1 - m_2 x_2 - m_3 x_3}{m_1 - m_2 - m_3} = \frac{M(0) - \frac{M}{16}(0) - \frac{M}{16}(3R)}{M - \frac{M}{16} - \frac{M}{16}}$$

$$= -\frac{3R}{14}$$

$$\bar{y} = \frac{m_1 y_1 - m_2 y_2 - m_3 y_3}{m_1 - m_2 - m_3} = \frac{M(0) - \frac{M}{16}(3R) - \frac{M}{16}(0)}{M - \frac{M}{16} - \frac{M}{16}}$$

$$= -\frac{3R}{14}$$

$$\text{Position vector of C.M.} = -\frac{3R}{14}(\hat{i} + \hat{j})$$

15. (c) This disc can be assumed to be made of a complete uniform disc and a square plate with the same negative mass density.

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{(\pi r^2)\sigma(0) + l^2(-\sigma)(r/2)}{\pi r^2\sigma + l^2(-\sigma)}$$

$$= \frac{-l^2 r}{2(\pi r^2 - l^2)} = \frac{-\frac{r^3}{2}}{2\left(\pi r^2 - \frac{r^2}{2}\right)} = \frac{-r}{4\left(\pi - \frac{1}{2}\right)}$$

DPP 1.2

Subjective Type

1. Initial velocity of centre of mass of (A + B) system is

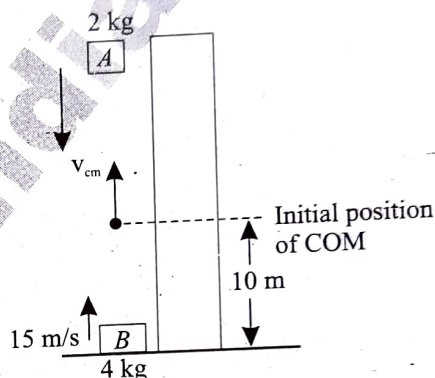
$$u_{cm} = \frac{1 \times 200 - 2 \times 85}{3} = 10 \text{ m/s upwards}$$

$\therefore$ Maximum height attained by centre of mass above its initial position is

$$H = \frac{u_{cm}^2}{2g} = \frac{100}{2 \times 10} = 5 \text{ metres}$$

2. Using $\frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2} = x_{cm}$

By putting values $x_{cm} = 10$ m from base.



$$\text{Initial velocity of COM} \Rightarrow \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 10 \text{ m/s upward}$$

Also, acceleration of COM is g thus using

$$v^2 = u^2 + 2as \text{ as for COM}$$

$$0 = 10^2 + 2 \times (-10) \times h_{\max}$$

$$h_{\max} = 5 \text{ m from the initial position of COM}$$

$$\text{Hence total height from base} = 10 + 5 \text{ m} = 15 \text{ m}$$

Single Correct Answer Type

3. (b) Friction force between the wedge and block is internal, i.e., will not change motion of COM. Friction force on the wedge by ground is external and causes COM to move towards right. Gravitational force (mg) on block brings it downward, hence COM comes down.

4. (c) Initial position of center of mass

$$\vec{r}_{cm} = \vec{r}_0 = (6\hat{i} + 14\hat{j} + 20\hat{k}) \text{ m}$$

Hence $x_{cm} = 6$ m, $y_{cm} = 14$ m and $z_{cm} = 20$ m

As there is no external force x on the system x -, y - and z -coordinates of center of mass should not change.

Let the coordinate of fourth part be (x, y, z)

$$x_{cm} = \frac{5 \times 10 + 3 \times 8 + 4 \times 4 + 6x}{18} = 6 \text{ m}$$

$$\Rightarrow x = 3 \text{ m}$$

$$y_{cm} = \frac{5 \times 20 + 3 \times 18 + 4 \times 10 + 6y}{18} = 10 \text{ m}$$

S.4 Mechanics II

$$\Rightarrow y = \frac{29}{3} \text{ m}$$

$$z_{\text{cm}} = \frac{5 \times 9 + 3 \times 25 + 4 \times 20 + 6z}{18} = 20 \text{ m}$$

$$z = \frac{80}{3} \text{ m}$$

$$\left[3, \frac{29}{3}, \frac{80}{3} \right]$$

Hence position vector of fourth part

$$\vec{r}_4 = \left(3\hat{i} + \frac{29}{3}\hat{j} + \frac{80}{3}\hat{k} \right) \text{ m}$$

$$\begin{aligned} 5. (d) \quad a_{\text{cm}} &= \frac{F_{\text{ext}}}{m_{\text{total}}} \\ &= \frac{0.2 \times (2+1) \times g}{2+1} = 2 \text{ m/s}^2 \end{aligned}$$

$$6. (b) \quad (m + m') \frac{x}{5} = mx + m'O$$

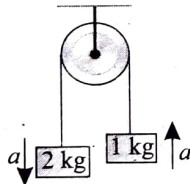
$$\therefore m + m' = 5, \quad m' = 4 \text{ m}, \quad \frac{m'}{m} = 4$$

7. (a) The centre of mass of man + disc shall always remain at the rest. Since the man is always at periphery of disc, the centre of disc shall always be at distance $R/3$ from centre of mass of two body system. Hence centre of disc moves in circle of radius $R/3$.

$$8. (b) \quad a = \frac{2g - 1g}{2+1} = \frac{10}{3} \text{ m/s}^2$$

$$\therefore a_{\text{COM}} = \frac{(2)(4) - (1)}{2+1} = \frac{a}{3} = \frac{10}{9} \text{ m/s}^2$$

(downwards)



$$\begin{aligned} \therefore S_{\text{COM}} &= \frac{1}{2} a_{\text{COM}} t^2 \\ &= \frac{1}{2} \left(\frac{10}{9} \right) (2)^2 = \frac{20}{9} \text{ m} \\ &= 2.22 \text{ m} \end{aligned}$$

9. (a) Acceleration of system

$$a = \frac{mg \sin 60^\circ - mg \sin 30^\circ}{2m}$$

Here m = mass of each block

$$\text{or } a = \left(\frac{\sqrt{3} - 1}{4} \right) g$$

$$\text{Now } \vec{a}_{\text{com}} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m}$$

Here $\vec{a}_1$ and $\vec{a}_2$ are $\left(\frac{\sqrt{3} - 1}{4} \right) g$ at the right angles.

$$\text{Hence, } |\vec{a}_{\text{com}}| = \frac{\sqrt{2}}{2} a = \left(\frac{\sqrt{3} - 1}{4\sqrt{2}} \right) g$$

10. (c) Let the tube displaced by x towards left, then

$$mx = m(R - x) \Rightarrow x = \frac{R}{2}$$

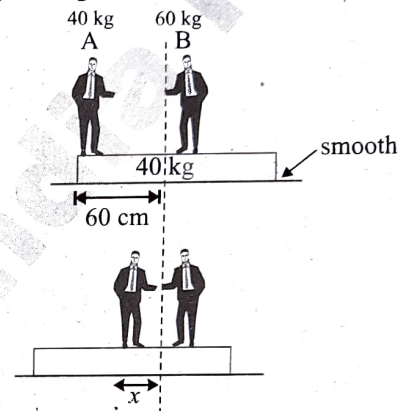
11. (b) Let the displacement of wedge be x (leftwards). Horizontal displacement of A and B with respect to wedge is $10 \cos 45^\circ$ or $5\sqrt{2}$ cm (rightwards) or the horizontal displacement of A and B with respect to ground is $(5\sqrt{2} - x)$ cm rightwards. The centre of mass of the whole system will not move in horizontal direction. Hence,

$$(2m)x = (5\sqrt{2} - x)(m + 2m)$$

$$\text{or } 5mx = 15\sqrt{2} \text{ m}$$

$$\text{or } x = 3\sqrt{2} \text{ cm}$$

12. (c) Taking the origin at the centre of the plank.



$$m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 = 0 \quad (\Delta x_{\text{CM}} = 0)$$

(Assuming the centres of the two men are exactly at the axis shown.)

$60(0) + 40(60) + 40(-x) = 0$, x is the displacement of the block.
 $\Rightarrow x = 60 \text{ cm}$, i.e., A and B meet at the right end of the plank.

$$m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3 = 0 \quad (\because \Delta x_{\text{CM}} = 0)$$

$$13. (b) \quad v_p = \frac{20}{5} = 4 \text{ m/s}$$

$$(x_{\text{cm}}) = 0$$

$$40(v_1 + 4) - 40(v_2 - 4) + 20 \cdot (4) = 0$$

$$40(v_1 - v_2 + 10) = 0$$

$$v_1 - v_2 = -10$$

If $v_1 = 10 \text{ m/s}$, $v_2 = \text{zero}$

$$v_1 = 20 \text{ m/s}, v_2 = 10 \text{ m/s}$$

$$v_1 = 15 \text{ m/s}, v_2 = 5 \text{ m/s}$$

14. (b) Initially x -coordinate of centre of mass is:

$$x_1 = \frac{(4M)(0) + M(5R)}{4M + M} = R \quad (1)$$

Let x_0 be the x -coordinate of shell when the small sphere reaches the other extreme position. Then finally x -coordinate of centre of mass is

$$x_f = \frac{(4M)(x_0) + M(x_0 - 5R)}{4M + M} = x_0 - R \quad (2)$$

All the surfaces are smooth; therefore, centre of mass will not move in x -direction.

$$\therefore x_i = x_f$$

$$\text{or } R = x_0 - R$$

$$\text{or } x_0 = 2R$$

15. (b) As both the balls are released simultaneously, at any instant before the lower balls reach the ground both have the same velocity; $v = gt$, i.e., ' v ' vs. ' t ' is a straight line graph.

$$V_{CM} = \frac{mv(t) + mv(t)}{2m} = v(t), \text{ (} v(t) \text{ being the instantaneous velocity.}$$

Just after the lower ball strikes ground and comes to rest:

$$V_{CM} = \frac{mv(t) + m(0)}{2m} = \frac{v(t)}{2}$$

i.e., the velocity suddenly drops to half its value.

Hence graphs (a) and (b) are chosen.

After collision:

$$a_{CM} = \frac{m(g) + m(0)}{m + m} = \frac{g}{2}$$

i.e., the slope (of $v-t$ curve) should decrease to half.
Hence (b) is the best option.

Multiple Correct Answers Type

16. (b, d)

Centre of mass will move in a vertical line and $v_1 \cos \theta_1 = v_2 \cos \theta_2$. Otherwise for any other values it will follow a parabolic path.

17. (a, c)

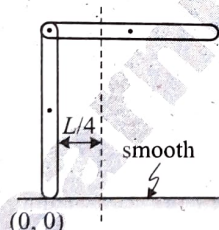
When the centre of mass remains at rest, it is possible that different individual forces do individual works, though the net resultant force is zero. As work is a scalar quantity, they get added up.

Also, $\Sigma F_{ext} = 0 \Rightarrow a_{CM} = 0$

18. (b, d)

Initially the centre of mass is at $\frac{L}{4}$ distance from the vertical rod.

$$\left(\text{As, } x_{cm} = \frac{m\left(\frac{1}{2}\right) + m(0)}{m + m} = \frac{L}{4} \right)$$



centre of mass does not move in x -direction as $\Sigma F_x = 0$.

After they lie on the floor, the pin joint should be at $L/4$ distance from the origin shown in order to keep the centre of mass at rest.

$\therefore$ Finally x -displacement of the pin is $\frac{L}{4}$ and y -displacement of the pin is obviously L .

$$\text{Hence net displacement} = \sqrt{L^2 + \frac{L^2}{16}} = \frac{\sqrt{17}L}{4}$$

DPP 1.3

Subjective Type

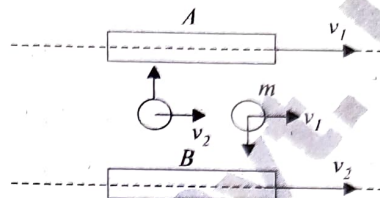
1. Since horizontal force on (wagon + body) system is zero. Then momentum is conserved in horizontal direction.

$$95 \times 10 - 5 (12.5 \cos 37^\circ) = 100 V$$

$$V = \frac{950 - 50}{100} = \frac{900}{100} = 9 \text{ m/s}$$

Single Correct Answer Type

2. (b) As no external force acting in horizontal direction, the acceleration of centre of mass will be zero.
As $v_1 > v_2$. By transferring equal mass:
 $mv_1 > mv_2$



Hence additional linear momentum will be transferred to train B than train A. Hence velocity of train A after this process will be less than the previous one.

3. (b) Applying momentum conservation,

$$(80)1 + 60(-2) = (80 + 60 + 100) v$$

$$v = \frac{-40}{240} = -\frac{1}{6} \text{ m/sec}$$

4. (a) Compression is maximum when both blocks move with the same velocity V .

By conservation of momentum

$$V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 5 \text{ m/s}$$

The change in K.E. = $k_f - k_i = -35 \text{ J}$

This is stored as spring PE.

$$\text{Therefore } \frac{1}{2} kx^2 = \Delta K \Rightarrow x = \sqrt{\frac{2\Delta K}{k}}$$

on solving $x = 0.25 \text{ m} = 25 \text{ cm}$

5. (d) Let the velocity of man after jumping be ' u ' towards right. Then speed of cart is $v - u$ towards left. From conservation of momentum

$$mu = 2m(v - u)$$

$$\therefore u = \frac{2v}{3}$$

Hence work done by man = change in KE of system

$$= \frac{1}{2} mu^2 + \frac{1}{2} 2m(v - u)^2$$

$$= \frac{1}{2} m \left(\frac{2v}{3} \right)^2 + \frac{1}{2} 2m \left(\frac{v}{3} \right)^2 = \frac{mv^2}{3}$$

6. (c) Let v be the final speed of block when it is at maximum height h . At that instant the speed of circular track shall also be v .



From the conservation of momentum

$$m\sqrt{2gR} = (m + 2m)v$$

(1)

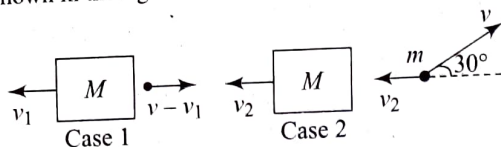
From the conservation of energy

$$\frac{1}{2} m (2gR) = \frac{1}{2} (m + 2m)v^2 + 2mg$$

(2)

Solving (1) and (2), we get $h = \frac{1}{3}R$

7. (b) Let mass of gun is M and that of shell is m . The two cases are shown in the figure as below.



Here v_1 and v_2 are the recoil speeds of the gun in two cases. Using conservation of linear momentum in horizontal direction in two cases:

$$m(v - v_1) = Mv_1$$

$$\therefore v_1 = \frac{mv}{M + m} \quad (1)$$

$$m(v \cos 30^\circ - v_2)$$

$$\therefore v_2 = \frac{\sqrt{3}mv}{2(M + m)} \quad (2)$$

From equations (1) and (2)

$$\frac{v_1}{v_2} = \frac{2}{\sqrt{3}}$$

8. (a) Let speed of block is v . Then from conservation of linear momentum in horizontal direction velocity of cylinder will be

$$2v \text{ in opposite direction (as } m = \frac{M}{2})$$

Now from conservation of mechanical energy we have

$$mgh = \frac{1}{2}Mv^2 + \frac{1}{2}m(2v)^2$$

Here $h = R - r = 1.0 \text{ m}$

Substituting the values, we get,

$$(1)(10)(1) = \frac{1}{2}(2)(v^2) + \frac{1}{2}(1)(4v^2)$$

$$\text{or } 3v^2 = 10$$

$$\therefore v = \sqrt{\frac{10}{3}} \text{ m/s}$$

9. (a) Momentum of the system remains conserved as no external force is acting on the system in horizontal direction.
 $\therefore (50 + 100)10 = 50 \times V + 100 \times 0 \Rightarrow V = 30 \text{ m/s}$ towards right, as boat is at rest. $V_{\text{boat}} = 30 \text{ m/s}$

10. (c) As shown in the figure, the components of momentum of one shell along initial direction and perpendicular to initial direction are $P_{1x} = 20\sqrt{3} \text{ Ns}$ and $P_{1y} = 20 \text{ Ns}$.

For momentum of the system to be zero in y -direction, P_{2y} must be 20 Ns . 2nd part of shell may or may not have momentum in x -direction

$$\therefore P_{2\text{min}} = 20 \text{ Ns}$$

11. (c) By conservation of linear momentum (consider man and plank as system)

$$mv + mv = m(-v) + mv'$$

(v' is the final speed of plank)

$$v' = 3v$$

Multiple Correct Answers Type

12. (b, c)

The resultant force can be accelerating or decelerating, hence the momentum can increase or decrease. Hence (a) is wrong.

$$\text{Since } F_{\text{net}} = M a_{\text{cm}}$$

$$\therefore a_{\text{cm}} \neq 0$$

hence v_{cm} must change

Hence (b).

In case of a circular motion of centre of mass about a point, the distance of centre of mass will remain constant. Hence (c).

Kinetic energy of some particles may increase and of some particles may decrease at the same time.

Comprehension Type

13. (a) Conserving linear momentum, $m_1 v_1 = m_2 v_2$
as $m_1 = m_2 = m \Rightarrow v_1 = v_2$

$$\text{Work-energy theorem} \Rightarrow mgh = 2\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow v = \sqrt{gh}$$

14. (c) Momentum conservation,

$$m \times \sqrt{gh} + m \times 0 = 2m \times v' \Rightarrow v' = \frac{\sqrt{gh}}{2}$$

Work-energy theorem,

$$= -mgh' = 2\left[\frac{1}{2}m\left(\frac{\sqrt{gh}}{2}\right)^2\right] - \frac{1}{2}m(\sqrt{gh})^2$$

$$\Rightarrow -mgh' = \frac{mgh}{4} - \frac{mgh}{2}$$

$$\Rightarrow -mgh' = -\frac{mgh}{4}$$

$$h' = \frac{h}{4}$$

15. (a) Momentum conservation,

$$m_1 v_1' + m v_2' = 2m \frac{\sqrt{gh}}{2}$$

Work-energy theorem

$$\frac{mgh}{4} = \left(\frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2\right) - \left[\frac{1}{2}2m\left(\frac{\sqrt{gh}}{2}\right)^2\right]$$

$$\Rightarrow v_1' = 0$$

$$v_2' = \sqrt{gh}$$

$\Rightarrow$ Block comes to rest and wedge moves with velocity $\sqrt{gh}$

DPP 1.4

Single Correct Answer Type

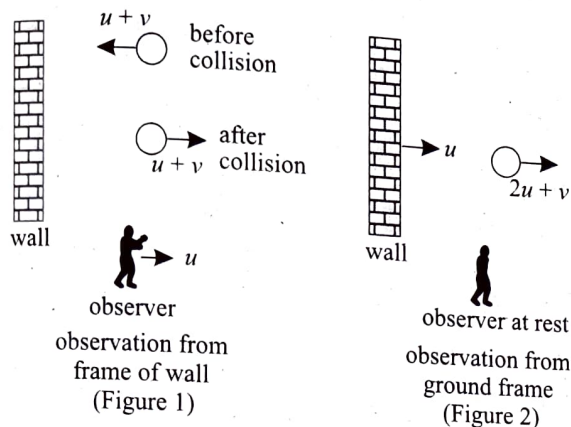
1. (a) $e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$

For elastic collision $e = 1$

[Velocity of separation] = [velocity of approach]
For inelastic collision $e < 1$

So [velocity of separation] < [velocity of approach]

2. (b) Nothing is mentioned about coefficient of restitution. Hence the only true statement is 'their final velocities may be zero.'
3. (b) From the frame of wall, the ball moves towards the wall with speed $u + v$ and after collision returns with velocity $u + v$ as shown in Figure 1. Then from ground frame, the velocity of ball after collision with wall is $2u + v$.



4. (b) Let the speeds of balls of mass m and $2m$ after collision be v_1 and v_2 as shown in the figure. Applying conservation of momentum

$$mv_1 + 2mv_2 = mu \text{ and } -v_1 + v_2 = \frac{u}{2}$$

Solving we get $v_1 = 0$ and $v_2 = \frac{u}{2}$

Hence the ball of mass m comes to rest and ball of mass $2m$ moves with speed.

$$t = \frac{2\pi r}{u/2} = \frac{4\pi r}{u}$$

5. (c) For first collision $v = 10 \text{ m/s}$. $t_1 = \frac{\pi(5)}{10} = \pi/2 \text{ sec}$

Velocity of sep = e . velocity of app.

$$v_2 - v_1 = \frac{1}{2}(10)$$

$$v_2 - v_1 = 5 \text{ m/s}$$

For second collision

$$\therefore t_2 = \frac{2\pi(5)}{5} = 2\pi$$

$$\therefore \text{Total time } t = t_1 + t_2 = \pi/2 + 2\pi$$

$$t = 2.5\pi$$

6. (d) With respect to elevator, initial velocity of ball = 5 m/s , downwards
Let velocity of ball just before striking the elevator be v ,
then $v^2 = 5^2 + 2 \times 10 \times 100$
 $= 25 + 2000 = 2025$
 $v = 45 \text{ m/s}$

Since the collision is elastic,

Velocity of approach = velocity of separation

$\therefore$ Velocity of separation = 45 m/s

$\therefore$ Velocity of ball after collision = 50 m/s (w.r.t ground)

$\therefore$ Velocity of ball after collision = 50 m/s (w.r.t ground)

7. (a) After the collision between 1 and 2, mass 1 stops and mass 2 proceeds to right with speed v_0 to collide with mass 3. The collision between masses 2 and 3 is perfectly inelastic. Hence 2 and 3 also come to rest after the collision.

8. (a) Let ' v ' be the initial velocity. Tangential velocity remains the same during collision and equal to $v \cos 60^\circ = v/2$. Let v' be the normal component of velocity after impact.

$$\text{In } \triangle OAB: \tan 60^\circ = \frac{v/2}{v'}$$

$$\Rightarrow v' = \frac{v}{2\sqrt{3}}$$

$$\text{Then } e = \frac{v'}{v \cos 30^\circ} = \frac{(v/2\sqrt{3})}{(\sqrt{3}v/2)} = \frac{1}{3}$$

9. (c) $e = \frac{v'_1 - v'_2}{u_2 - u_1} = \frac{v_1 - v_2}{u - 3u}$

$$\text{or } e \times 2u = v_2 - v_1$$

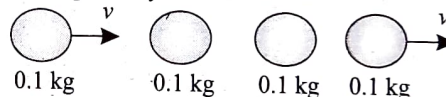
10. (a) In case of perfectly inelastic collision the velocity of combined mass will be $\frac{v}{2}$

$$\therefore \text{KE} = \frac{1}{2} \left(\frac{v^2}{4} \right)$$

$$(0.2) = \frac{1}{2} (0.2) \left(\frac{v^2}{4} \right)$$

$$\text{or } v = 2\sqrt{2} \text{ m/s}$$

In case of perfectly elastic collision



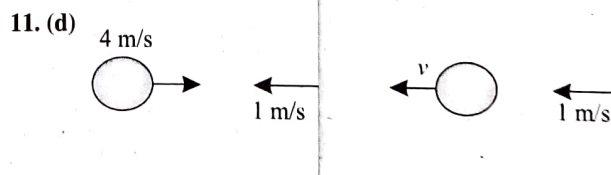
First particle stops and the second acquires the same velocity v .

$$\text{KE} = \frac{1}{2} mv^2$$

$$\text{or } 0.2 = \frac{1}{2} (0.1) v^2$$

$$\text{or } v = 2 \text{ m/s}$$

Therefore, minimum value of v is 2 m/s in case of perfectly elastic collision and maximum value of v is $2\sqrt{2} \text{ m/s}$ in case of perfectly collision.



Before collision

After collision

Let v be the velocity of ball after collision,

Collision is elastic

$$\therefore e = 1$$

or relative velocity of separation

= relative velocity of approach

$$\text{or } v = 6 \text{ m/s}$$

(away from the wall)

12. (b) Let u be the velocity of ball before collision. Speed of ball after collision will become.

$$v = \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}} u$$

S.8 Mechanics II

∴ Fraction of K.E. lost in collision

$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - \left(\frac{v}{u}\right)^2 = 1 - \frac{5}{8} = \frac{3}{8}$$

13. (a) Let the masses are unequal



Before Collision

After Collision

From the conservation of linear momentum;

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$\text{or } m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

$$\text{or } m_1 = m_2$$

i.e., mass of both the particles should be same

$$\text{Secondly, } e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$$

$$= \frac{v_1' - v_2'}{v_1 - v_2} = 1; \text{ i.e., } e = 1 \text{ or collision is elastic.}$$

14. (a) In an elastic oblique collision, both particles move at right angles after collision if mass of both the particles are the same provided one particle is at rest before collision.

Multiple Correct Answers Type

15. (b, d)

In elastic collision, the velocities are exchanged if masses are the same.

∴ after the collision

$$V_C = 0$$

$$V_A = v$$

Now the maximum compression will occur when both the masses A and B move with the same velocity.

$$\therefore mv = (m + m)V \text{ (for system of A - B and spring)}$$

$$\therefore V = \frac{v}{2}$$

$$\therefore \text{KE of the A-B system} = \frac{1}{2} \times 2m \left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}$$

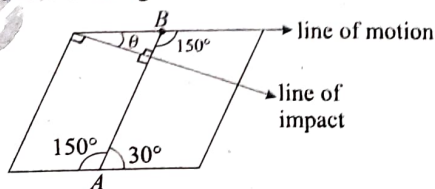
And at the time of maximum compression,

$$\frac{1}{2}mv^2 = \frac{1}{2} \times 2m \left(\frac{v}{2}\right)^2 + \frac{1}{2}KX_{\max}^2 \therefore X_{\max} = v\sqrt{\frac{m}{2K}}$$

DPP 1.5

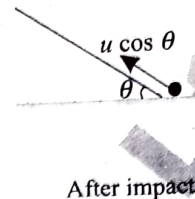
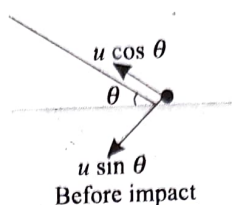
Single Correct Answer Type

1. (b) As shown in the figure



$$\theta + 90^\circ = 150^\circ \therefore \theta = 60^\circ$$

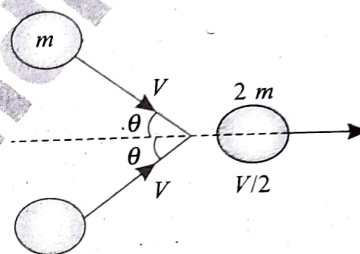
2. (b) Just before the particle transfers to inclined surface, we resolve its velocity along and normal to the plane.



For the trajectory of the particle to sharply change from the horizontal line to the inclined line, the impact of the particle with inclined plane should reduce the $u \sin \theta$ component of velocity to zero. Hence the particle moves up the incline with speed $u \cos \theta$.

Hence as θ increases, the height to which the particle rises shall decrease.

3. (a) Initial angle between the velocities is 2θ and after the collision, combined mass moves along the bisector line. Conserving momentum along final direction of motion,

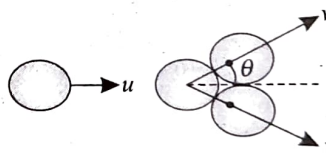


$$2m v \cos \theta = 2m \cdot \frac{v}{2}$$

$$\Rightarrow \theta = 60^\circ$$

$$\Rightarrow 2\theta = 120^\circ$$

4. (c)



$$\sin \theta = \frac{r}{2r} = \frac{1}{2}$$

$$\therefore \theta = 30^\circ$$

From conservative of linear momentum

$$mu = 2mv \cos 30^\circ$$

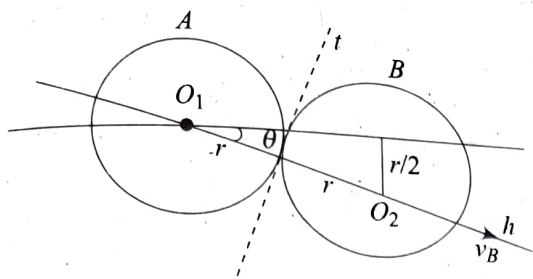
$$\text{or } v = \frac{u}{\sqrt{3}} \quad (1)$$

Now $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$ in common normal direction.

$$\text{Hence, } e = \frac{v}{u \cos 30^\circ} = \frac{u/\sqrt{3}}{u/\sqrt{3}/2} = \frac{2}{3}$$

5. (a) Normal force by the rough surface and frictional force are impulsive in nature. Hence conservation of linear momentum is applicable for m only along common tangent.

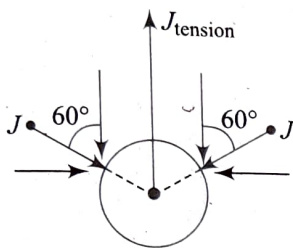
$$6. (a) \sin \theta = \frac{r/2}{2r} = \frac{1}{4}$$



As collision is elastic velocity along common normal, it will be interchanged.

$$\text{Hence } v_B = v \cos \theta = \frac{v \cdot \sqrt{15}}{4}$$

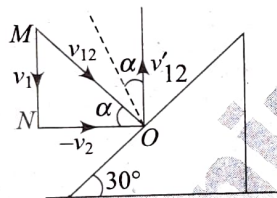
$$7. (c) J_{\text{tension}} = 2J \cos 60^\circ$$



and $J = mu$

$$J_{\text{tension}} = 2(mu) \cdot \frac{1}{2} = mu$$

8. (b) In the figure $\vec{v}_{12}$ = velocity of ball w.r.t. wedge before the collision and $\vec{v}'_{12}$ = velocity of ball w.r.t. wedge after the collision, which must be in vertically upward direction as shown. In elastic collision, $\vec{v}_{12}$ and $\vec{v}'_{12}$ will make equal angle (say α) with the normal to the plane.



We can show that $\alpha = 30^\circ \therefore \angle MON = 30^\circ$

$$\text{Now } \frac{v_1}{v_2} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

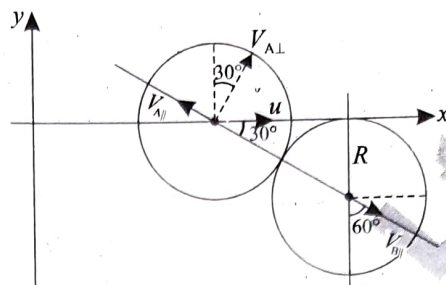
Multiple Correct Answers Type

9. (c, d)

Sphere A moving with velocity v has a component $v\sqrt{3}/2$ along the line joining the centres of the spheres at the time of collision and another component $v/2$ perpendicular to the previous direction. After collision the component along the line will interchange, i.e., B will move with $v/2$ velocity, i.e., 4 m/s along the line joining the centres and A moves with $v\sqrt{3}/2$ velocity at perpendicular direction to B.

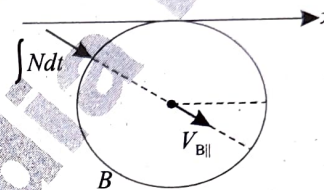
Comprehension Type

10. (a) During collision, forces act along the line of impact. As collision is elastic and both the balls have the same mass, velocities are exchanged along the line of impact. Therefore ball B moves with velocity $V_{B||}$, that is equal to $u \cos 30^\circ$. Ball A moves perpendicular velocity $V_{A\perp}$, that is equal to $u \cos 60^\circ$. Along the line of impact, ball A does not have any velocity after the collision. Therefore velocity of ball A in vector form after the collision



$$\begin{aligned} &= V_{A\perp} \cos 60^\circ i + V_{A\perp} \cos 30^\circ j \\ &= (u \cos 60^\circ) \cos 60^\circ i + (u \cos 60^\circ) \cos 30^\circ j \\ &= 4 \cdot \frac{1}{2} \cdot \frac{1}{2} i + 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} j = (i + \sqrt{3}j) \text{ m/s} \end{aligned}$$

11. (c) Using impulse-momentum equation for ball B



$$\int N dt = \vec{p}_f - \vec{p}_i \text{ and as } \vec{p}_i = 0$$

$$\int N dt = \vec{p}_f$$

$$= (mu \cos 30^\circ) \cos 30^\circ i - (mu \cos 30^\circ) \cos 60^\circ j$$

$$= m \cdot 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} i - m \cdot 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} j$$

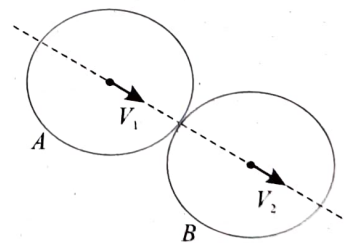
$$= (3mi - \sqrt{3}mj) \text{ kg } \frac{\text{m}}{\text{s}}$$

12. (b) Suppose V_2 is velocity of ball B along the line of impact and V_1 is velocity of ball A along the line of impact, after the collision, as shown.

Then $\frac{1}{2}$ (Velocity of approach) = Velocity of separation

$$\frac{1}{2} \left[\frac{\sqrt{3}}{2} \cdot u \right] = V_2 - V_1 \quad (1)$$

Conserving momentum along the line of impact



$$m \cdot \frac{\sqrt{3}}{2} u = m \cdot V_2 + m V_1 \quad (2)$$

Solving and using $u = 4 \text{ m/s}$

$$V_2 = \frac{3\sqrt{2}}{2} \text{ m/s}$$

$$\vec{V}_2 = \frac{3\sqrt{3}}{2} \cos 30^\circ i - \frac{3\sqrt{3}}{2} \cos 60^\circ j$$

$$= \left(\frac{9}{4} i - \frac{3\sqrt{3}}{4} j \right) \text{ m/s}$$

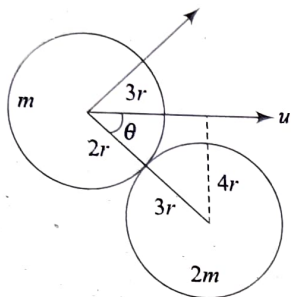
S.10 Mechanics II

13. (d) 14. (a) 15. (b)

Tangential component = $u \sin \theta = 4u/5$

Along normal velocity of m before collision:

$$u_1 = u \cos \theta = \frac{3u}{5}$$



Along normal: $mv_1 + 2mv_2 = mu_1$

$$\frac{v_2 - v_1}{u_1} = 1$$

Solve to get $v_1 = -\frac{u}{5}$, $v_2 = \frac{2u}{5}$

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CHAPTER 2

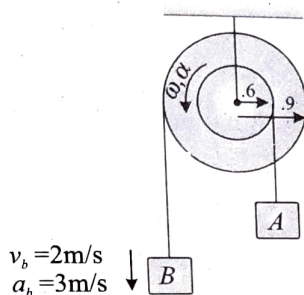
DPP 2.1

Subjective Type

$$1. v_b = r_b \omega \Rightarrow \omega = \frac{2}{.9}$$

$$v_a = r_a \omega = .6 \times \frac{2}{.9} = \frac{4}{3} \text{ m/s}$$

Also



$$a_b = r_b \cdot a \alpha \Rightarrow \alpha = \frac{a_b}{r_b} = \frac{3}{.9} = \frac{10}{3} \text{ r/s}^2$$

$$a_a = r_a \cdot \alpha \Rightarrow \alpha_0 = .6 \times \frac{3}{.9} = 2 \text{ m/s}^2$$

- (i) Applying equation $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ for pulley from $t=0$ to $t=2$ sec.

$$\theta = \frac{2}{.9} \times 2 + \frac{1}{2} \cdot \frac{10}{3} \cdot 4$$

$$= \frac{40}{9} + \frac{40}{6} = \frac{80+120}{18} = \frac{200}{18}$$

$$\text{No. of revolution} = \frac{\theta}{2\pi} = \frac{200}{18} \times \frac{1}{2\pi} = \frac{50}{9\pi}$$

- (ii) Displacement of A

$$\Rightarrow s = ut + \frac{1}{2} at^2 = \frac{4}{3} \times 2 + \frac{1}{2} \times 2 \times 4 = \frac{20}{3} \text{ m}$$

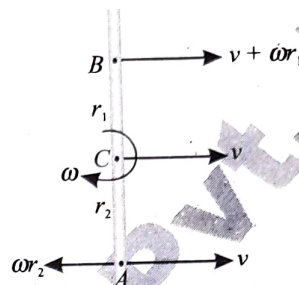
2. Time in which C.M. reaches its highest point = 1 sec (from $v = u + at$, putting $v = 0$, $u = +10$, $a = 10 \text{ m/sec}^2$) after projection angular velocity will not change as the torque of external forces is zero.

In 1 sec, the rod will rotate by an angle $= \omega t = \frac{\pi}{2} \times 1 \text{ rad}$. The rod will be vertical with point A at the lowest point.

$$\therefore a_A = g - \frac{\omega^2 L}{2} = 10 - \frac{\pi^2 4}{4 \times 2} = 5 \text{ m/sec}^2$$

Single Correct Answer Type

3. (c) Suppose a rod is having angular velocity ω about point C.



Choose two points A and B as shown in the figure velocity of B w.r.t. A $= (v + \omega r_1) - (v - \omega r_2)$

$$\Rightarrow V_{B/A} = \omega(r_1 + r_2)$$

Angular velocity of B w.r.t. $\omega_{BA} = \frac{v_{B/A}}{AB}$

$$\omega_{BA} = \frac{\omega(r_1 + r_2)}{r_1 + r_2} = \omega$$

4. (d) Given $a_A = 2\alpha = 5 \text{ m/s}^2$

$$\Rightarrow \alpha = \frac{5}{2} \text{ rad/s}^2$$

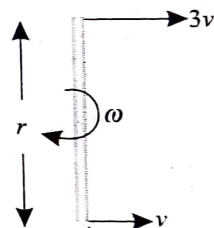
$$\Rightarrow a_B = 1.(\alpha) = 5/2 \text{ m/s}^2$$

5. (d) $\omega_{rod} = \omega_{point} = \left(\frac{v_{rel}}{r} \right)$, v_{rel} being the velocity of one point w.r.t. other.

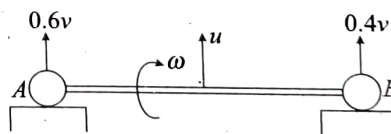
$$= \frac{3v - v}{r} \text{ and 'r' being the distance}$$

between them.

$$= \frac{2v}{r}$$



6. (a) If v is the velocity by which balls strike the bases, then their velocities after collision are $0.6v$ and $0.4v$ as shown. ' u ' is velocity of COM just after collision and ω is the angular velocity acquired by the rod.



$$\text{For end A: } 0.6v = u + \frac{l}{2} \cdot \omega \quad (i)$$

$$\text{For end B: } 0.4v = u - \frac{l}{2} \cdot \omega \quad (ii)$$

Subtracting (i) and (ii) we get

$$0.2v = \omega l \Rightarrow \omega = \frac{v}{5l}$$

Now using $[v^2 = u^2 + 2as]$

$$v^2 = 0 + 2 \times 10 \times 0.2 = 4 \Rightarrow v = 2 \text{ m/s}$$

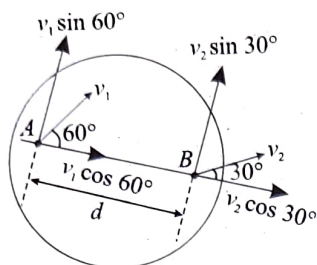
$$\omega = \frac{2}{5 \times 1} = \frac{2}{5} \text{ rad/sec}$$

S.12 Mechanics II

7. (d) For rigid body separation between the two point remains the same.

$$v_1 \cos 60^\circ = v_2 \cos 30^\circ$$

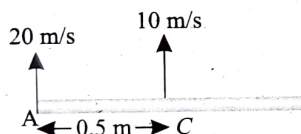
$$\frac{v_1}{2} = \frac{\sqrt{3}v_2}{2} \Rightarrow v_1 = \sqrt{3}v_2$$



$$\omega_{\text{disc}} = \frac{v_2 \sin 30^\circ - v_1 \sin 60^\circ}{d} = \frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}$$

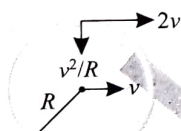
$$= \frac{v_2 - \sqrt{3} \times \sqrt{3}v_2}{2d} = \frac{2v_2}{2d} = \frac{v_2}{d} \Rightarrow \omega_{\text{disc}} = \frac{v_2}{d}$$

8. (a) Angular velocity $\omega = \frac{20-10}{0.5} = 20 \text{ rad/sec}$



9. (c) Radius of Curvature = $\frac{(\text{velocity})^2}{\text{Normal acceleration}}$

$$= \frac{(2v)^2}{v^2/R} = 4R$$



Multiple Correct Answers Type

10. (c, d)

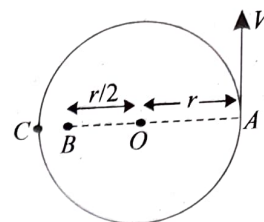
All points in the body, in plane perpendicular to the axis of rotation revolve in concentric circles. All points lying on circle of the same radius have the same speed (and also the same magnitude of acceleration) but different directions of velocity (also different directions of acceleration)

Hence there cannot be two points in the given plane with the same velocity or with the same acceleration.

As mentioned above, points lying on circle of same radius have same speed.

Angular speed of body at any instant w.r.t. any point on body is the same by definition.

11. (a, c)



$$\frac{\omega_{AO}}{\omega_{AB}} = \frac{v/r}{v/(3r)} = \frac{v/r}{2v/3r} = \frac{3}{2}$$

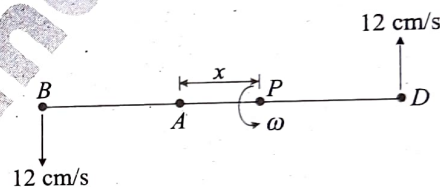
$$\omega_{AC} = \frac{v}{2r} = \frac{\omega}{2}$$

Note: Construct a circle of that radius from the given point about which the angular velocity is to be detected.

Comprehension Type

- Sol. 12. (b) 13. (a)

Suppose point P is the instantaneous centre of rotation. Then



For point B: $12 = (2+x)\omega$ (i)

For point C: $12 = (4-x)\omega$ (ii)

Solving (i) and (ii), we get $x = 1 \text{ cm}$ (to the right of A) and $\omega = 4 \text{ rad/sec}$

Velocity of center of pulley A, $v_A = x\omega$

$$v_A = 1 \times 4 = 4 \text{ cm/s}$$

- Sol. 14. (c) 15. (a)

Load B is moving down with velocity v. Let angular velocity of pulley system be ω .

$$v_B = R\omega \Rightarrow 2 = 0.9 \times \omega$$

$$\Rightarrow \omega = \frac{20}{9} \text{ rad/sec}$$

Also the acceleration of load B is $a_B = 3 \text{ m/s}^2$ (downward)

$$\text{Hence } a_B = R\alpha \Rightarrow 3 = 0.9\alpha \Rightarrow \alpha = \frac{10}{3} \text{ rad/sec}$$

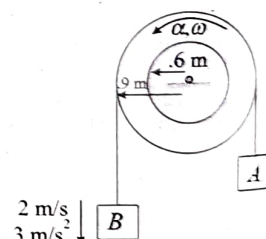
The initial velocity of load 'A'

$$v_A = r\omega = 0.6 \times \frac{20}{9} = \frac{4}{3} \text{ m/s}$$

and acceleration of block B

$$a_B = r\alpha = 0.6 \times \frac{10}{3} = 2 \text{ m/s}^2$$

$$\text{Angle rotated by pulley, } \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$



$$\Rightarrow \theta = \frac{20}{9} \times 2 + \frac{1}{2} \times \frac{10}{3} \times 4 = \frac{100}{9} \text{ rad}$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{100}{2\pi \times 9} = \frac{50}{9\pi}$$

For displacement of load A using, $s = ut + \frac{1}{2}at^2$

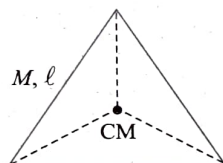
$$s_A = \frac{4}{3} \times 2 + \frac{1}{2} \times 2 \times 4 = \frac{20}{3} \text{ m}$$

DPP 2.2

Subjective Type

1. MI of the system w.r.t an axis $\perp$ to plane and passing through one corner

$$\begin{aligned} &= \frac{ML^2}{3} + \frac{ML^2}{3} + \left[\frac{ML^2}{12} + M \left(\frac{\sqrt{3}L}{2} \right)^2 \right] \\ &= \frac{2ML^2}{3} + \left[\frac{ML^2}{12} + \frac{3ML^2}{4} \right] \\ &= \frac{2ML^2}{3} + \frac{10ML^2}{12} = \frac{3ML^2}{3} = \frac{18ML^2}{12} = \frac{3}{2}ML^2 \end{aligned}$$



$$\text{Now } \frac{3}{2}ML^2 = 3k^2 \Rightarrow k = \frac{L}{\sqrt{2}}$$

2. By given question

Area of square = Area of rectangular plate

$$\Rightarrow c^2 = a \times b \Rightarrow c^2 = ab$$

Now by definition

$$(a) \frac{I_{xR}}{I_{xS}} = \frac{b^2}{c^2}$$

From the diagram $b < c$

$$\Rightarrow \frac{I_{xR}}{I_{xS}} = \left(\frac{b}{c} \right)^2 < 1 \Rightarrow I_{xR} < I_{xS}$$

$$(b) \frac{I_{yR}}{I_{yS}} = \frac{a^2}{c^2} \text{ as } a > c$$

$$\frac{I_{yR}}{I_{yS}} = \left(\frac{a}{c} \right)^2 > 1$$

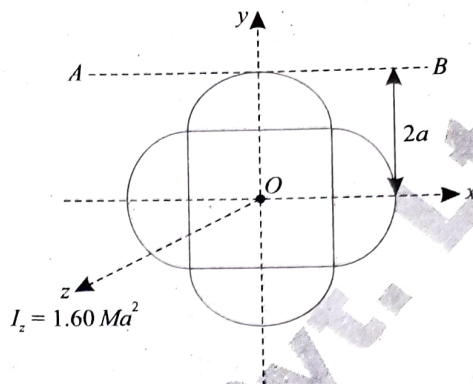
$$(c) I_{zR} - I_{zS} \propto (a^2 + b^2 - 2c^2) \\ = a^2 + b^2 - 2ab = (a - b)^2 \quad [\because c^2 = ab]$$

$$\Rightarrow (I_{zR} - I_{zS}) > 0 \Rightarrow \frac{I_{zR}}{I_{zS}} > 1$$

3. Using $\perp$ axis theorem $I_z = I_x + I_y$.

But $I_x = I_y$, hence $I_z = 2I_x$

$$\text{Hence } 2I_x = 1.6Ma^2 \Rightarrow I_x = 0.8Ma^2$$



Now using parallel axis theorem

$$I_{AB} = I_y + M(2a)^2 = 0.8Ma^2 + 4Ma^2 = 4.8Ma^2$$

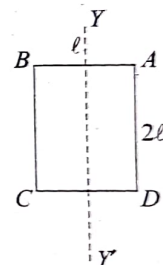
Single Correct Answer Type

4. (b) In the given diagrams, when the small piece Q is removed and glued to the centre of the plate, the mass comes closer to the z-axis; hence, moment of inertia decreases.
5. (d) Moment of inertia of the elliptical disc should be less than that of a circular disc having radius equal to the major axis of the elliptical disc.
- Hence (d).

$$6. (c) I \text{ (about } YY') = \frac{ml^2}{12}$$

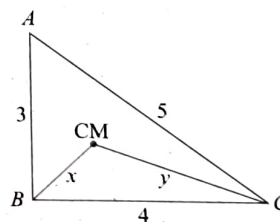
Using parallel axis theorem:

$$\text{(about AD)} \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$



7. (a) Moment of inertia is more when mass is farther from the axis. In case of axis BC, mass distribution is closest to it and in case of axis AB mass distribution is farthest. Hence

$$I_{BC} < I_{AC} < I_{AB} \Rightarrow I_P > I_B > I_H$$



$$I_C = I_{CM} + my^2 = I'_B - mx^2 + my^2$$

Here I'_B is moment of inertia of the plate about an axis perpendicular to it and passing through B.

$$\Rightarrow I_C = I'_B + m(y^2 - x^2) = I_P + I_R + m(y^2 - x^2)$$

It means $I_C > I_P + I_B$ also $I_C > I_P$

$$\therefore I_C > I_P > I_B > I_H$$

8. (a) Divide the ring into infinitely small lengths of mass dm_i . Even though mass distribution is non-uniform, each mass dm_i is at the same distance R from origin.
- $\therefore$ MI of ring about z-axis is
- $$= dm_1 R^2 + dm_2 R^2 + \dots + dm_n R^2 = MR^2$$
9. (d) $I = I_0 + 6I'$
- I_0 is the moment of inertia of central disc and I' is the moment of inertia of rest of the each disc about specified axis.

S.14 Mechanics II

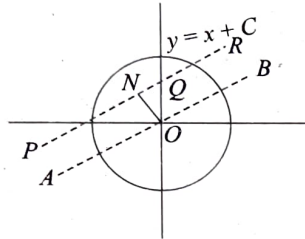
By parallel axes theorem

$$I = \frac{mr^2}{2} + 6(I_{cm} + md^2) = \frac{mr^2}{2} + 6\left[\frac{mr^2}{2} + m(2r)^2\right]$$

$$= \frac{mr^2}{2} + \frac{6mr^2}{2} + 24mr^2 = \frac{55mr^2}{2}$$

10. (b) $I_{PQR} = I_{AOB} + M \cdot (ON)^2$

$$I_{PQR} = \frac{1}{4}MR^2 + M \cdot \left(\frac{c}{\sqrt{2}}\right)^2$$



But $I_{PQR} = \frac{1}{2}MR^2 \therefore c = \pm \frac{R}{\sqrt{2}}$

Hence (b) is correct.

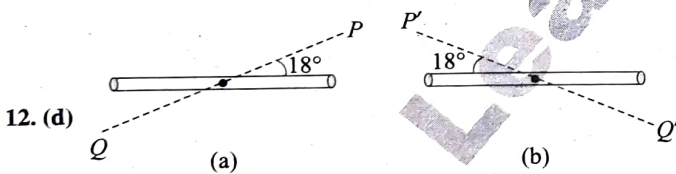
11. (d) Taking mass of plate $m = \frac{ma^2}{6} \times 2 = \frac{ma^2}{3}$

Then MI of two plates through which the axis is passing
MI of 4 plates having symmetrical position from the axis

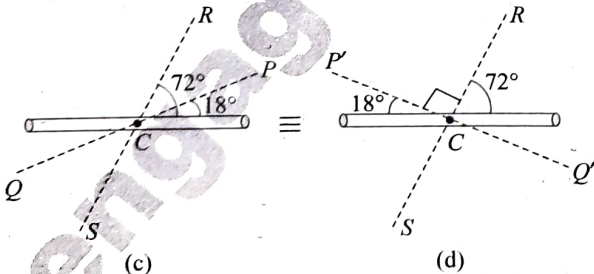
$$= 4 \times \left[\frac{ma^2}{12} + m\left(\frac{a}{2}\right)^2 \right] = 4 \times \left[\frac{ma^2}{3} \right]$$

$$\text{Total MI} = \frac{4ma^2}{3} + \frac{ma^2}{3} = \frac{5ma^2}{3}$$

$$\text{Using } \frac{M}{6} = m = \text{MI} = \frac{5Ma^2}{18}$$



The MI of rod about axis PQ [Figure (a)] and MI of rod about axis P'Q' [Figure (b)] are the same by symmetry.



$$\therefore I_{PQ} + I_{RS} = I_{PQ} + I_{RS} = \frac{ml^2}{12}$$

by perpendicular axis theorem.

13. (b) Let m_1 = mass of the square plate of side 'a'
and m_2 = mass of the square of side 'a/2'

Then $m_1 = \sigma \left(\frac{a}{2}\right)^2$ and $m_2 = \sigma(a)^2$ (σ being the areal density)
and $m_2 - m_1 = M$.

$$\Rightarrow I = \frac{m_2 a^2}{6} - \left\{ \frac{m_1 (a/2)^2}{6} + m_1 \left(\frac{a}{4}\right)^2 \right\}$$

$$= \frac{\sigma a^4}{6} - \left\{ \frac{\sigma (a/2)^4}{6} + \sigma \left(\frac{a}{2}\right)^2 \cdot \left(\frac{a}{4}\right)^2 \right\}$$

$$\Rightarrow I = \sigma a^4 \left\{ \frac{27}{12 \times 16} \right\}$$

$$\text{Also } M = \sigma \left(1 - \frac{1}{4}\right) a^2 \Rightarrow \sigma = \frac{4M}{3a^2}$$

$$\Rightarrow \left(\frac{4M}{3a^2}\right) \cdot a^4 \left\{ \frac{27}{12 \times 16} \right\} \Rightarrow I = \frac{3Ma^2}{16}$$

Multiple Correct Answers Type

14. (b, d)

(a) Theorem of perpendicular axes is applicable only for laminar (plane) objects. Thus, option (a) is false.

(b) As $z' \parallel z$ and distance between them

$$= a \frac{\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$$

Now, by theorem of parallel axes

$$I_{z'} = I_z + m \left(\frac{a}{\sqrt{2}}\right)^2 = I_z + \frac{ma^2}{2}$$

Hence, choice (b) is true.

(c) z'' is not parallel to z hence, theorem of parallel axis cannot be applied. Thus, option (c) is false.

(d) As x and y -axes are symmetrical.

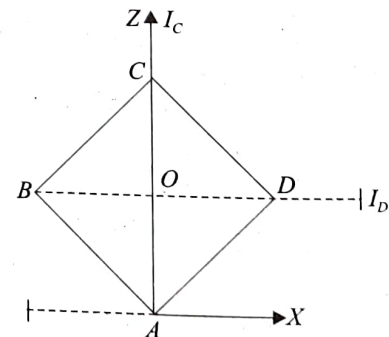
$$\text{Hence, } I_x = I_y$$

Thus, option (d) is true.

15. (a, c)

When rotating in $(x-z)$ in $(x-z)$ plane.

$$I_D = I_C = \frac{Ml^2}{6}$$



I_O (axis passing through O and perpendicular to I_D and I_C)

$$I_O = \frac{Ml^2}{6} + \frac{Ml^2}{6} = \frac{Ml^2}{3}$$

$$I_A = I_O + M(OA)^2$$

$$\Rightarrow I_1 = \frac{Ml^2}{3} + M \left(\frac{l}{\sqrt{2}}\right)^2 = \frac{5}{6}Ml^2$$

When plate rotates in (x-z) plane, axis of rotation will be perpendicular to this plane passing through A. When plate rotates by falling towards (x-y) plane, then axis of rotation will be along line IOx , as shown, which is parallel to line BD along which M . $IM I_D$ is known.

$$I_2 = I_D + M \left(\frac{l}{\sqrt{2}} \right)^2 = \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{4}{6} Ml^2$$

$$\frac{I_1}{I_2} = \frac{5}{4}$$

DPP 2.3

Concept Based

1. No, not necessarily.

Given, $\sum F_i \neq 0$

The sum of torques about a certain point O ,

$$\sum r_i \times F_i = 0$$

The sum of torques about any other point O'

$$\sum (r_i - a) \times F_i = \sum r_i \times F_i - a \times \sum F_i$$

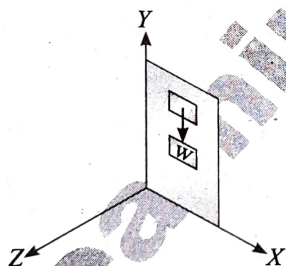
Here, the second term need not vanish.

Therefore, sum of all the torques about any arbitrary point need not be zero necessarily.

2. Consider the diagram, where weight of the door acts along the negative y-axis.

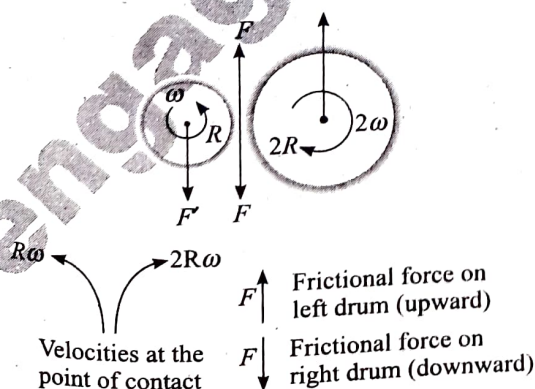
A force can produce torque only along a direction normal to itself as $\tau = r \times F$. So, when the door is in the xy-plane, the torque produced by gravity can only be along $\pm z$ direction never about an axis passing through y-direction.

Hence, the weight will not produce any torque about y-axis.

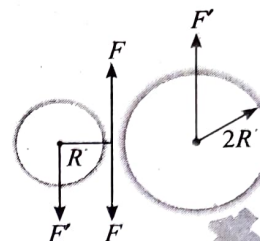


Subjective Type

3. (a) Consider the situation shown below, we have shown the frictional forces.



- (b) $F' = F = F''$ where F' and F'' are external forces through support.
 $\Rightarrow F_{\text{net}} = 0$ (one each cylinder)
 External torque = $F \times 3R$ (anticlockwise)

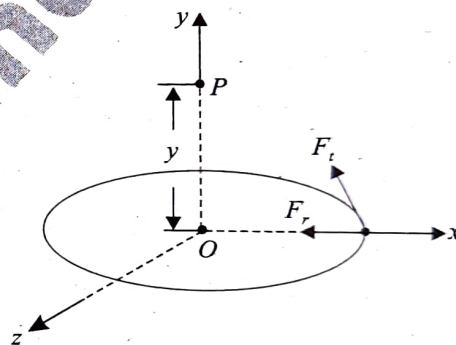


- (c) Let ω_1 and ω_2 be final angular velocities of smaller and bigger drums respectively (anticlockwise and clockwise respectively). Finally, there will be no friction.

Hence, $R\omega_1 = 2R\omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = 2$

4. (a) At any instant the tangential and radial forces acting on the particle are given as, $F_t = ma_t = ma$ (because $a_t = 0$ as given)

And $F_r = ma_r = \frac{mv^2}{r}$ respectively, as shown in the figure.



Then, the net torque about P is given as the sum of torques produced by F_t and F_r about P which can be given as

$$\vec{\tau}_P = F_t y \downarrow + F_r r \uparrow$$

$$= -F_r y \hat{k} + F_t r \hat{j} \text{ where } F_r = \frac{mv^2}{r} \text{ and } F_t = ma$$

$$\text{Then } \vec{\tau}_P = -\frac{mv^2 y}{r} \hat{k} + mar \hat{j}$$

$$= \frac{m}{r} (ar^2 \hat{j} - v^2 y \hat{k})$$

Hence, the magnitude of the torque is $\tau_P = \frac{m}{r} \sqrt{a^2 r^4 + v^4 y^2}$

- (b) $\vec{\tau}_x = may \hat{i}$, $\vec{\tau}_y = mar \hat{j}$ and $\vec{\tau}_z = -\frac{mv^2 y}{r} \hat{k}$

Single Correct Answer Type

5. (a) Block will be tipped about point O,

$$M_O = (F \cos 30^\circ) a$$

$$= (F \sin 30^\circ) + \frac{mgL}{2}$$

$$F = \frac{mgL}{(a\sqrt{3} - L)}$$

For $F > 0$, $a\sqrt{3} > L$

$$a \geq L/\sqrt{3}$$

6. (a) $F - F_1 = ma$
or $F_1 = F - ma$
 $\tau_C = 0$ so that rod may move translationally

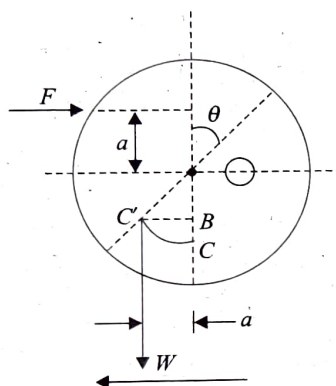
$$F\left(\frac{L}{2} - a_0\right) = F_1(2a_0)$$

$$\frac{FL}{2} = a_0(2F_1 + F) = a_0(3F - 2ma)$$

$$L = \frac{2a_0(3F - 2ma)}{F}$$

For $L > 0$, $3F > 2ma$, or $a < 3F/2m$

7. (d) The sphere is in stable equilibrium. It will come back to original position after rotation. Rotating Moment = Fa . For stable equilibrium,



Restoring moment = Rotating moment

Since $F = W$, $W_a = Fa$ (i)

Let O be the centre and C and C' be the positions of CG before and after rotation.

$$OC = OC' \text{ and } C'B = a$$

[From eq. (i)]

In $\triangle OBC'$ and $C'B \csc \theta = a \csc \theta$

Multiple Correct Answers Type

8. (a, b, c, d)

We know that torque on a system of particles

$$\tau = r \times F = F \sin \theta \hat{n} \quad (i)$$

where θ is angle between r and F , and $\hat{n}$ is a unit vector perpendicular to both r and F .

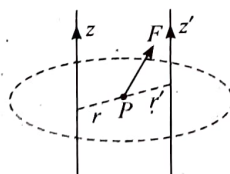
- (a) When forces act radially, $\theta = 0$ hence $|\tau| = 0$ [from Eq. (i)]
(b) When forces are acting on the axis of rotation, $r = 0$, $|\tau| = 0$ [from Eq. (i)]
(c) When forces acting parallel to the axis of rotation $\theta = 0^\circ$, $|\tau| = 0$ [from Eq. (i)]
(d) When torque by forces are equal and opposite, $\tau_{\text{net}} = \tau_1 - \tau_2 = 0$

9. (b, c)

(a) Consider the adjacent diagram, where $r > r'$

Torque τ about z -axis $\tau = r \times F$ which is along $\hat{k}$

(b) $\tau' = r' \times F$ which is along $-\hat{k}$



- (c) $|\tau|_z = Fr_\perp =$ magnitude of torque about z -axis where $r_\perp$ is perpendicular distance between F and z -axis.

Similarly, $|\tau|_{z'} = Fr'_\perp$

Clearly $r_\perp > r'_\perp \Rightarrow |\tau|_z > |\tau|_{z'}$

- (d) We are always calculating resultant torque about a common axis.

Hence, total torque $\tau \neq \tau + \tau'$ because τ and τ' are not about common axis.

10. (a, d)

The F.B.D. of cylinder is as shown. In equilibrium

$\Rightarrow T = Mg =$ tension in string

In equilibrium, net force on cylinder is zero.

$\therefore$ Torque is the same about any axis.

$\Rightarrow$ Torque on cylinder about any point is zero.

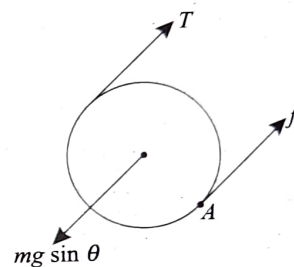
$$\tau_A = Mg \cdot 2R - mg \sin \theta R = 0$$

$\therefore M = \frac{m \sin \theta}{2}$. Hence for only one value of M

cylinder can remain in equilibrium. $\Rightarrow A$ is true and B is false

When the cylinder rolls up the incline, sense of rotation of cylinder about the center of mass is clockwise. Hence $T > f \Rightarrow C$ is false.

When the cylinder rolls down the incline, sense of rotation of cylinder about center of mass is anticlockwise. Hence $T < f \Rightarrow D$ is True.

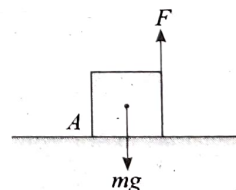


Matching Column Type

11. Consider the below diagram

Moment of the force F about point A , $\tau_1 = F \times a$ (anticlockwise)

Moment of weight mg of the cube about point A , $\tau_2 = mg \times \frac{a}{2}$ (clockwise)



Cube will not exhibit motion, if $\tau_1 = \tau_2$

($\because$ In this case, both the torque will cancel the effect of each other)

$$F \times a = mg \times \frac{a}{2} \Rightarrow F = \frac{mg}{2}$$

Cube will rotate only when $\tau_1 > \tau_2$

$$F \times a > mg \times \frac{a}{2} \Rightarrow F > \frac{mg}{2}$$

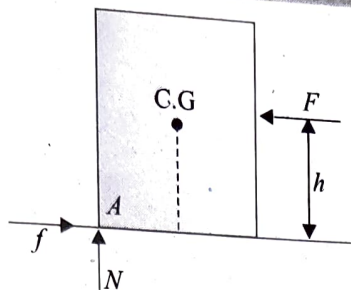
Let normal reaction is acting at $\frac{a}{3}$ from point A , then

$$mg \times \frac{a}{3} = F \times a \text{ or } F = \frac{mg}{3} \text{ (For no motion)}$$

When $F = \frac{mg}{4}$ which is less than $\frac{mg}{3}$ ($F < \frac{mg}{3}$), there will be no motion.

Comprehension Type

12. (a) Normal reaction N will pass through CG of the block in absence of any external force. With the application of horizontal force F , right part of block will try to lift. N will pass through the edge at the moment of tipping over about this edge, A. Greatest h will cause tipping.



$$\Sigma F_x = 0 \text{ will give } f - F = 0$$

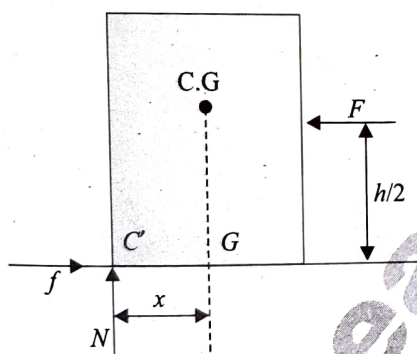
$$\text{or } F = f = \mu mg$$

$$\Sigma F_y = 0 \text{ will give } N - mg = 0 \text{ or } N = mg$$

$$\Sigma \tau_A = 0 \text{ will give } h - mg \frac{b}{2} \text{ or } h = \frac{mgb}{2F}$$

$$\text{or } h = \frac{mgb}{2\mu mg} = \frac{b}{2\mu}$$

13. (a) In this case, there is no state of impending tipping. Assume resultant of frictional force and normal reaction passes through C' , which is at a horizontal distance x from CG



$$(\mu N) \left(\frac{h}{2} \right) - N \cdot x = 0$$

$$x = \frac{\mu h}{2}$$

14. (b) For impending tipping N will act at the left edge. Equation for impending tipping.

$$\Sigma \tau = 0 = Fh_{\max} - mg \frac{b}{2} = 0$$

$$h_{\max} = \frac{mgb}{2F}$$

Equation of impending sliding.

$$\Sigma F_x = 0 = -F + \mu N = 0$$

$$\Sigma F_y = 0 = -mg + N = 0$$

From eqs. (i) and (ii), equation for impending sliding and tipping both

$$h_{\max} = \frac{b}{2\mu}$$

DPP 2.4

Single Correct Answer Type

$$1. (b) \quad Tr = \frac{mr^2}{2} \alpha_1 \quad (1)$$

$$Tr = \frac{mr^2}{2} \alpha \quad (2)$$

$$\alpha_1 = \alpha \quad (3)$$

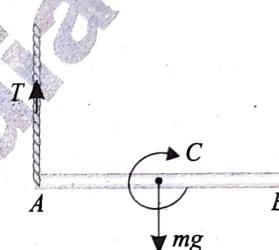
From (1) and (2)

accⁿ. of point $b = \text{acc}^n$. of point a

$$r\alpha_1 = a_{cm} - r\alpha \quad (4)$$

Hence $2r\alpha = a_{cm}$

$$2. (c) \text{ Applying Newton's law on center of mass } mg - T = ma \quad (i)$$



Writing $\tau = I\alpha$ about center of mass

$$T \frac{l}{2} = \frac{ml^2}{12} \alpha \quad (ii)$$

$$\text{Also } a = \frac{l}{2} \alpha \quad (iii)$$

From (i), (ii) and (iii)

$$\alpha = \frac{3g}{2l}$$

3. (c) Applying Newton's law on centre of mass O
 $Mg - T = ma$ ($a = \text{acceleration of centre of mass}$)

$\tau = I\alpha$, about center of mass

$$T \frac{R}{2} = \frac{MR^2}{2} \cdot \alpha$$

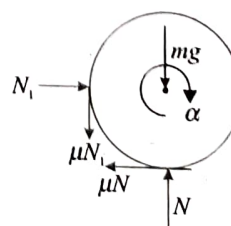
$$\text{Also } a = \frac{R}{2} \alpha$$

$$\text{From above equations } T = \frac{2Mg}{3}$$

4. (b) For linear motion, apply Newton's laws

$$N_1 - \mu N = 0 \mu_1 = \mu N$$

$$\mu N_1 + mg = N$$



(1)

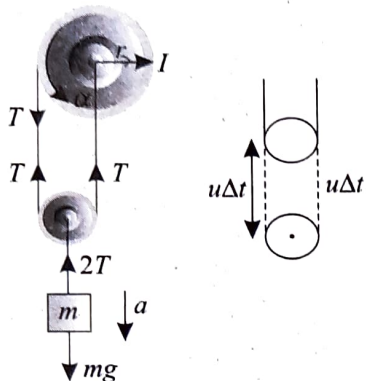
(2)

For rotation motion,

$$(\mu N - \mu N_1) \cdot r = \frac{mr^2}{2} \alpha$$

$$\frac{\mu mg}{1 - \mu^2} - \frac{\mu^2 mg}{1 - \mu^2} = \frac{mr}{2} \alpha \Rightarrow \alpha = \frac{2\mu g}{r(1 + \mu)}$$

5. (c) Let at any moment angular velocity of pulley be ω and downward linear velocity of light pulley is u . In small duration Δt , fixed pulley shall release length of rope = $r\omega\Delta t$. In some duration light pulley goes down by $u \cdot \Delta t$.



Thus $r\omega\Delta t = 2u\Delta t$

Differentiating equation (i) both sides

$$r \frac{d\omega}{dt} = 2 \frac{du}{dt} \Rightarrow r \cdot \alpha = 2a \Rightarrow \alpha = \frac{2a}{r}$$

Now, equation of linear motion for block

$$mg - 2T = ma$$

Equation of rotation for pulley

$$Tr = I\alpha$$

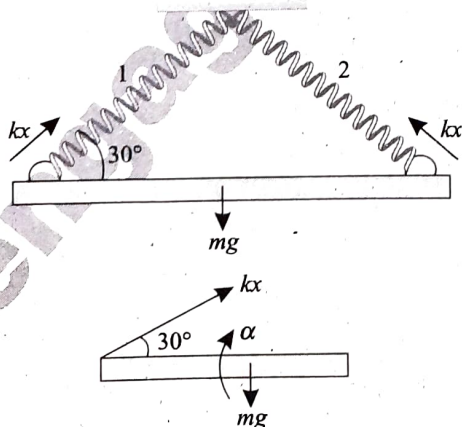
$$T \cdot r = I \cdot \frac{2a}{r} \Rightarrow 2T = \frac{4aI}{r^2}$$

$$T \cdot r = I \cdot \frac{2a}{r} \Rightarrow 2T = \frac{4aI}{r^2}$$

$$mg = ma + \frac{4aI}{r^2}$$

$$a = \frac{mg}{m + \frac{4I}{r^2}} \Rightarrow a = \frac{mr^2 g}{mr^2 + 4I}$$

6. (c)



m equilibrium

$$2kx \sin 30^\circ = mg$$

$$kx = mg$$

When spring is snapped
equation of rotation motion about COM.

$$kx \cdot \sin 30^\circ \cdot \frac{l}{2} = I\alpha$$

$$= \frac{ml^2}{12} \alpha \Rightarrow \alpha = \frac{3g}{l}$$

7. (d) About point O , Torque $\tau = I\alpha$

$$Mg \left(\frac{L}{2} \cos \theta \right) = \frac{ML^2}{3} \alpha \Rightarrow \frac{3g}{2L} \cos \theta = \alpha$$

Initially centripetal acceleration of point P is zero

$$\left(\because a_c = \frac{v^2}{r} = \frac{0}{r} = 0 \right)$$

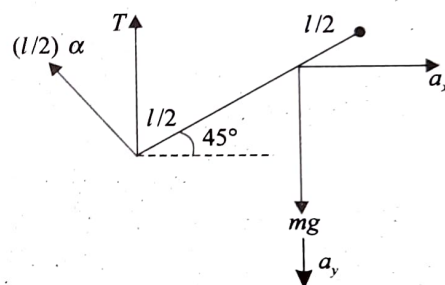
Acceleration of point P is $\sqrt{a_c^2 + a_t^2}$

$$= a_t = L\alpha = \frac{3}{2}g \cos \theta$$

8. (a) Let a_x and a_y be the linear acceleration of centre of mass and α is the angular acceleration of rod about CM

$$\alpha = \frac{\tau}{I} = \frac{T \cdot (l/2) \cos 45^\circ}{(ml^2/12)} = \frac{3\sqrt{2}T}{ml}$$

$$a_x = 0, a_y = \frac{mg - T}{m}$$



Tangential acceleration = $\frac{l}{2} \cdot \alpha$

$$a_y = \frac{l}{2} \cdot \alpha \cos 45^\circ$$

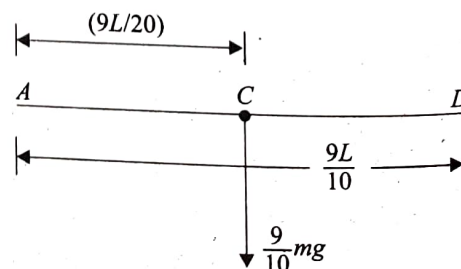
$$\text{So, } \frac{mg - T}{m} = \frac{l}{2} \cdot \frac{3\sqrt{2}T}{ml} \cdot \frac{1}{\sqrt{2}} = \frac{3T}{2m}$$

$$T = \frac{2}{5}mg$$

9. (a) Torque = $\tau = \frac{9}{10}mg \left(\frac{9}{20}L \right) = I\alpha = \frac{m}{3} \left(\frac{9}{10}L \right)^2 \alpha$

$$\alpha = \frac{3g}{2L}$$

Acceleration, $a_{CM} = \alpha (AC)$



$$= \frac{3g}{2L} \left(\frac{9L}{20} \right) = \frac{27g}{40}$$

$$\text{Now, } \frac{9}{10}mg - N_A = ma_{\text{CM}}$$

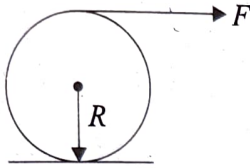
$$= m \cdot \frac{27g}{40} \text{ or } N_A = \frac{9}{40}mg$$

$$10. (d) \tau = F \times R = I\alpha$$

In one full rotation,

$$\theta = 2\pi = \frac{1}{2}\alpha t^2$$

$$t^2 = \frac{4\pi}{\alpha} = \frac{4\pi I}{FR}$$



Linear acceleration, $a = F/M$

Distance travelled in one rotation
 $= 2\pi R$ (given)

$$= \frac{1}{2}at^2 = \frac{1}{2} \frac{F}{M} \cdot \frac{4\pi I}{FR} = \frac{2\pi I}{MR}$$

It gives $I = MR^2$

It is a hollow cylinder.

M.I. of the rod w.r.t axis of rotation

$$I_x = I_{\text{cm}} + \frac{mL^2}{16} = \frac{mL^2}{12} + \frac{mL^2}{16} = \frac{7mL^2}{48}$$

$$(i) Mg \cdot \frac{L}{4} = I \cdot \alpha$$

$$Mg \cdot \frac{L}{4} = \frac{7mL^2}{48} \cdot \alpha \Rightarrow \alpha = \frac{12g}{7L}$$

$$\text{Hence } a_B = R\alpha = \frac{3L}{4} \cdot \frac{12}{7L} = \frac{9g}{7} \downarrow$$

$$(ii) a_{\text{cm}} = \frac{L}{4} \cdot \alpha = \frac{3g}{7}$$

Also apply equation of motion on center of mass

$$mg - N = m \left(\frac{3g}{7} \right)$$

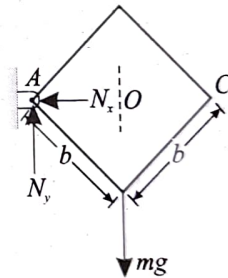
$$N = mg - \frac{3mg}{7} = \frac{4mg}{7} \uparrow$$

Sol. 13. (b) 14. (a) 15. (d)

$$Mg(b/\sqrt{2}) = I\alpha, I = \frac{Mb^2}{6} + M \left(\frac{b}{\sqrt{2}} \right)^2$$

$$\Rightarrow I = \frac{Mb^2}{6} + \frac{Mb^2}{2} = \frac{Mb^2}{2} \left(1 + \frac{1}{3} \right)$$

$$I = \frac{2Mb^2}{3}$$



$$\text{Hence } \frac{Mgb}{\sqrt{2}} = \frac{2Mb^2}{3} \alpha \quad \alpha = \frac{3g}{2\sqrt{2}b}$$

$$\text{Acc}^n \text{ of corner } C = \sqrt{b^2 + b^2} \alpha = \frac{3g}{2}$$

Velocity of O is zero. So $N_x = 0$

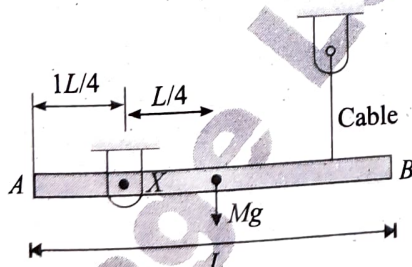
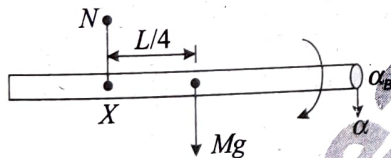
$$Mg - N_y = M \frac{b}{\sqrt{2}} \alpha = \frac{b}{\sqrt{2}} \left(\frac{3g}{2\sqrt{2}b} \right) = \frac{3Mg}{4}$$

$$\therefore N_y = \frac{Mg}{4}$$

Comprehension Type

Sol. 11. (a) 12. (b)

Taking torques w.r.t 'x'



CHAPTER 3

DPP 3.1

Subjective Type

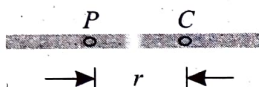
1. We have $\vec{v}_C = \frac{\vec{v}_1 + \vec{v}_2}{2}$ and $\omega = \frac{|\vec{v}_{12}|}{l} = \frac{|\vec{v}_1 - \vec{v}_2|}{l}$

Then put v_C and ω in the expression

$$K = \frac{1}{2}mv_C^2 + \frac{1}{2}I_C\omega^2$$

2. (i) Total kinetic energy of the rod

$$K_{\text{rod}} = \frac{1}{2}I_P\omega^2 = \frac{1}{2}(I_C + mr^2)\omega^2$$



Where $I_C = \frac{ml^2}{12}$ and $r = \frac{l}{4}$

$$K_{\text{rod}} = \frac{1}{2} \left[\frac{ml^2}{12} + m \left(\frac{l}{4} \right)^2 \right] \left(\sqrt{\frac{g}{l}} \right)^2 = \frac{7mgl}{96}$$

- (ii) Rotational kinetic energy

$$K_{\text{rotation}} = \frac{1}{2}I_C\omega^2 = \frac{1}{2} \left(\frac{ml^2}{12} \right) \left(\sqrt{\frac{g}{l}} \right)^2$$

$$\Rightarrow K_{\text{rotation}} = \frac{mgl}{24}$$

- (iii) Translational kinetic energy

$$K_{\text{trans}} = \frac{1}{2}mv_C^2$$

Speed of center of mass.

$$v_C = \omega \cdot \frac{l}{4} = \left(\sqrt{\frac{g}{l}} \right) \cdot \frac{l}{4} = \frac{1}{4}\sqrt{gl}$$

$$\text{Hence } K_{\text{trans}} = \frac{1}{2}m \left(\frac{1}{4}\sqrt{gl} \right)^2$$

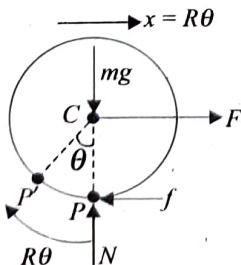
$$\Rightarrow K_{\text{trans}} = \frac{1}{32}mgl$$

3. The work done by all forces is given as

$$W = \int \vec{\tau}_C \cdot d\vec{\theta} + \int \vec{F}_{\text{ext}} \cdot d\vec{S}_C$$

Where $\tau_C = fR$ because F , mg and N cannot produce torque about C as they pass through it and $F_{\text{ext}} = (F - f)$.

$$\text{Then, } W = \int_0^\theta fR d\theta + \int_0^x (F - f) dx$$

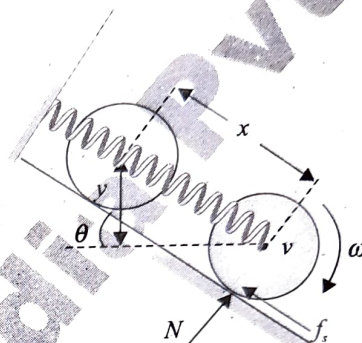


This gives $W = f(R\theta - x) + Fx$

In pure rolling, $x = R\theta$; hence the work done by friction is zero.

Then, the total work done is given by $W = Fx$

4. Let the sphere rolls down such that its centre of mass falls through a vertical distance y during its displacement x .



$$y = x \sin \theta$$

Then, the change in potential energy is $\Delta U = \Delta U_g + \Delta U_s$

Since the centre of mass falls, $\Delta U_g = -mgy - mgx \sin \theta$ (because $y = x \sin \theta$)

As the spring is connected at the centre of the sphere, it is elongated through a length x . Hence

$$\Delta U_s = \frac{k}{2}x^2$$

$$\text{Then, } \Delta U = -mgx \sin \theta - \frac{k}{2}x^2$$

The sphere gains a KE which is given as $\Delta K = \frac{mv^2}{2} \left(1 + \frac{k^2}{R^2} \right)$.

$$\text{where } \frac{k^2}{R^2} = \frac{2}{5}$$

$$\text{Then, } \Delta K = \frac{7}{10}mv^2$$

Since the forces f_x and N do not perform any work, we have $\Delta U + \Delta K = 0$

$$\text{Substituting } \Delta U = -mgx \sin \theta - \frac{kx^2}{2}$$

$$\text{and } \Delta K = \frac{7}{10}mv^2$$

In the above equation and simplifying the factors, we have

$$v = \sqrt{\frac{10}{7}gx \sin \theta - \frac{5k}{7m}x^2}$$

Single Correct Answer Type

5. (b) Translation kinetic energy $K_{\text{trans}} = \frac{1}{2}mv_c^2$

Rotational kinetic energy

$$K_{\text{rot}} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}(mk^2) \left(\frac{v_2}{R} \right)^2 = \frac{1}{2}mv_c^2 \left(\frac{k^2}{R^2} \right)$$

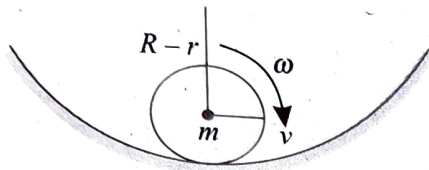
Hence total kinetic energy,

$$K_{\text{total}} = K_t + K_{\text{rot}}$$

$$K_{\text{total}} = \frac{1}{2}mv^2 \left(1 + \frac{k^2}{R^2} \right)$$

$$\text{Then, } \frac{K_t}{K_{\text{total}}} = \frac{1}{1 + \frac{k^2}{R^2}} \Rightarrow \frac{K_{\text{total}}}{K_{\text{trans}}} = \left(1 + \frac{k^2}{R^2} \right)$$

6. (b) When the sphere reaches its lowest position it loses gravitational potential energy by ΔU and gains a kinetic energy by ΔK , where $\Delta U = -mg(R-r)$



$$\text{and } \Delta K = \frac{1}{2}mv_c^2 \left(1 + \frac{k^2}{R^2} \right) = \frac{1}{2}mv^2 \left(1 + \frac{2}{5} \right) = \frac{7}{10}mv^2$$

According to conservation of energy, $\Delta U + \Delta K = 0$

$$\text{or } -mg(R-r) + \frac{7}{10}mv^2 = 0$$

$$\text{or } v = \sqrt{\frac{10}{7}g(R-r)}$$

7. (d) The total KE is $K = K_{\text{rod}} + K_{\text{hollow sphere}} + K_{\text{solid sphere}}$

$$\text{Where } K_{\text{rod}} = \frac{1}{2}mv^2$$

Since the CM of each sphere moves with a velocity

$$v_c = \frac{v}{2},$$

$$K_{\text{hollow sphere}} = \frac{1}{2}mv_c^2 \left(1 + \frac{k^2}{R^2} \right) = \frac{1}{2}m \left(\frac{v}{2} \right)^2 \left(1 + \frac{2}{3} \right) = \frac{5mv^2}{24}$$

$$K_{\text{solid sphere}} = \frac{1}{2}m \left(\frac{v}{2} \right)^2 \left(1 + \frac{2}{5} \right) = \frac{7}{40}mv^2$$

Using the above four equations,

$$K = \frac{1}{2}mv^2 + \frac{5}{24}mv^2 + \frac{7}{40}mv^2 = \frac{53}{60}mv^2$$

8. (c) For (rod + particle) system:

$$\frac{1}{2} \left(\frac{ml^2}{3} \right) \left(\frac{v^2}{l^2} \right) + \frac{1}{2}mv^2 = 2mg \left(\frac{3l}{4} \right)$$

[Since, COM will finally reach a height $2 \left(\frac{3l}{4} \right)$]

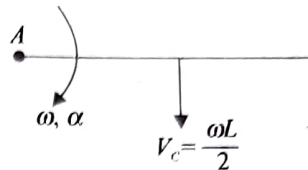
$$\Rightarrow v = \sqrt{4.5gl}$$

Comprehension Type

9. (c) By conservation of mechanical energy.

$$mg \frac{L}{2} = \frac{1}{2} \left(\frac{mL^2}{3} \right) \omega^2 \Rightarrow \omega = \left(\frac{3g}{L} \right)^{1/2}$$

$$10. (c) V_{\text{COM}} = \frac{\omega L}{2} = \frac{1}{2}(3gL)^{1/2}$$



11. (c) N_x = horizontal reaction at A
= Centripetal force of centre of mass

$$= \frac{mV_c^2}{(L/2)} = \frac{2m}{L} \cdot \frac{3gL}{4} = \frac{3mg}{2}$$

12. (d) $mg - N_y = ma_y$
But a_y = acceleration of COM in y-direction

$$= \frac{L}{2} \alpha$$

$$N_y = mg - \frac{mL\alpha}{2}$$

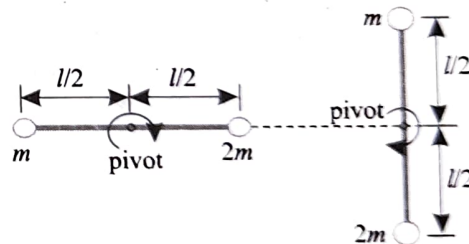
$$\text{But } \alpha = \frac{\tau_A}{I_A} = \frac{mg(L/2)}{mL^2/3} = \frac{3g}{2L}$$

$$\text{So, } N_y = mg - \frac{mL}{2} \cdot \frac{3g}{2L} = \frac{mg}{4}$$

$$13. (a) N = \sqrt{N_x^2 + N_y^2} = mg \sqrt{\frac{37}{16}}$$

14. (a), 15. (b)

Decrease in PE = Increase in rotation K.E



$$\Rightarrow 2mg \cdot \frac{l}{2} - mg \cdot \frac{l}{2} = \frac{1}{2}I \cdot \omega^2$$

$$\Rightarrow mg \cdot \frac{l}{2} = \frac{1}{2} \left(2m \frac{l^2}{4} + m \cdot \frac{l^2}{4} \right) \omega^2$$

$$\frac{mg l}{2} = \frac{1}{2} \cdot \frac{3ml^2}{4} \cdot \omega^2 = \frac{3ml^2}{8} \omega^2$$

$$\omega = \sqrt{\frac{4g}{3l}} \text{ and } v = r\omega = \frac{l}{2} \sqrt{\frac{4g}{3l}} = \sqrt{\frac{gl}{3}}$$

DPP 3.2

Subjective Type

1. If rigid body kinematics, we learnt that the velocity of the centre of mass of the rod can be given as $\vec{v}_C = \frac{\vec{v}_1 + \vec{v}_2}{2}$

$$\vec{v}_{\text{rod}} = m\vec{v}_C = \frac{m}{2}(\vec{v}_1 + \vec{v}_2)$$

Summing up the momenta of other two particles with that of the rod, we have the momentum of the system, which can be given as $\vec{P} = \vec{P}_{\text{rod}} + 2m\vec{v}_1 + m\vec{v}_2$

$$\text{Substituting } \vec{v}_{\text{rod}} = \frac{m}{2}(\vec{v}_1 + \vec{v}_2)$$

$$\vec{P} = 2.5m\vec{v}_1 + 1.5m\vec{v}_2$$

2. Let the common angular velocity of the system is ω .

(a) Yes, the law of conservation of angular momentum can be applied. Because, there is no net external torque on the system of the two discs.

External forces, gravitation and normal reaction act through the axis of rotation, hence, produce no torque.

(b) By conservation of angular momentum

$$\begin{aligned} L_f &= L_i \\ \Rightarrow I\omega &= I_1\omega_1 + I_2\omega_2 \\ \Rightarrow \omega &= \frac{I_1\omega_1 + I_2\omega_2}{I} = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} \quad (\because I = I_1 + I_2) \end{aligned}$$

$$\begin{aligned} (c) \quad K_f &= \frac{1}{2}(I_1 + I_2) \frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)^2} \\ &= \frac{1}{2} \frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)} \end{aligned}$$

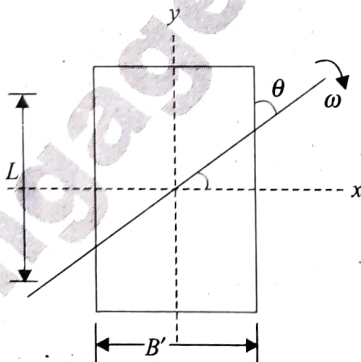
$$K_i = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$$

$$\Delta K = K_f - K_i = -\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 < 0$$

(d) Hence, there is loss in KE of the system. The loss in kinetic energy is mainly due to the work against the friction between the two discs.

Single Correct Answer Type

$$3. (a) \quad I_z = I_x + I_y = \frac{M}{12}(L^2 + B^2)$$



$$L_z = I_z \omega_z = 0$$

(since $\omega_z = 0$)

$$4. (a) \quad L_x = I_x \omega_x = \frac{ML^2}{12} \omega \sin \theta$$

$$L_y = I_y \omega_y = \frac{MB^2}{12} \omega \cos \theta$$

$$L_z = 0; L = L_x \hat{i} + L_y \hat{j} + L_z \hat{k}$$

$$= \frac{M\omega}{12} (L^2 \sin^2 \theta \hat{i} + B^2 \cos^2 \theta \hat{j})$$

5. (a) Angular momentum about point D is conserved.

$$mva = I_D \omega$$

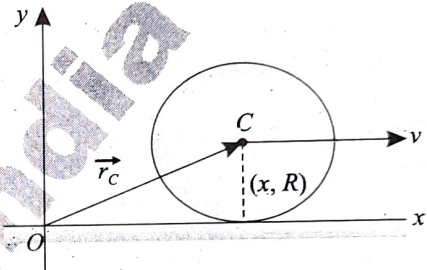
$$\text{But } I_D = \frac{5}{6} Ma^2. \text{ So, } \omega = \frac{6mv}{5Ma}$$

6. (d) $L = (mg \sin \theta) R t$

$$\text{Since, } \tau = \frac{\Delta L}{\Delta t}, \text{ so } L = \tau(\Delta t)$$

The curve between L and time t will be a straight line.

7. (b) The angular momentum of the disc about O is



$$\vec{L}_O = m\vec{r}_C \times \vec{v}_C + I_C \vec{\omega}$$

$$= m(x\hat{i} + R\hat{j}) \times v\hat{i} + \frac{1}{2} mR^2 \left(\frac{3v}{R} \hat{k} \right)$$

$$= m(x\hat{i} + R\hat{j}) \times v\hat{i} + \frac{1}{2} mR^2 \left(\frac{3v}{R} \hat{k} \right) m v R (\hat{j} \times \hat{i}) + \frac{3}{2} m v R \hat{k}$$

$$-m v R \hat{k} + \frac{3}{2} m v R \hat{k} = \frac{m v R}{2} \hat{k}$$

8. (a) As no external torque acts on the system, angular momentum should be conserved.

Hence $I\omega = \text{constant}$

where I is moment of inertia of the system and ω is the angular velocity of the system.

From Eq. (i) $I_1 \omega_1 = I_2 \omega_2$

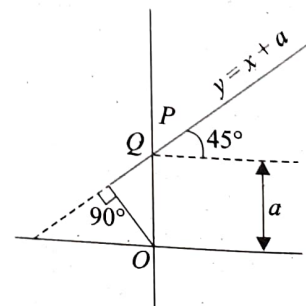
(where ω_1 and ω_2 are angular velocities before and after jumping)

$$\Rightarrow I\omega = \frac{I}{2} \times \omega_2$$

(as mass reduced to half; hence, moment of inertia is also reduced to half)

$$\Rightarrow \omega_2 = 2\omega$$

9. (c) $\angle POQ = 45^\circ$



$$\text{So, } OQ = a \cos 45^\circ = a/\sqrt{2}$$

$$L = mv(OQ) = mva/\sqrt{2}$$

10. (a) $I_1\omega_1 = I_2\omega_2$
 Since, men move towards middle of turn table I_2 decreases
 hence ω_2 increases.

$$\begin{aligned} \therefore \Delta K &= \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}I_2\omega_2^2 \\ &= \frac{1}{2}I_1\omega_1^2 \left[1 - \frac{I_2}{I_1} \cdot \frac{\omega_2^2}{\omega_1^2} \right] \quad \left\{ \frac{\omega_2}{\omega_1} > 1 \right\} \\ &= \frac{1}{2}I_1\omega_1^2 \left[1 - \frac{\omega_2}{\omega_1} \right] < 0 \end{aligned}$$

So kinetic energy increases.

Multiple Correct Answers Type

11. (a, c)

For a general rotational motion, where axis of rotation is not symmetric. Angular momentum L and angular velocity ω need not be parallel. For a general translational motion momentum $p = mv$ hence p and v are always parallel.

12. (a, b, c)

As $\Sigma \tau = 0$; angular momentum and linear momentum remain conserved.

As the two balls will move radially out, I changes. In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.

Comprehension Type

$$13. (c) \quad \frac{1}{2}I(2\omega)^2 = \frac{1}{2}Kx_1^2$$

$$\text{and } \frac{1}{2}(2I)\omega^2 = \frac{1}{2}Kx_2^2$$

$$\text{On solving } \frac{x_1}{x_2} = \sqrt{2}$$

14. (a) From conservation of angular momentum

$$(I + 2I)\omega' = I(2\omega) + (2I)\omega$$

$$\omega' = \frac{4}{3}\omega$$

Angular Impulse = change in angular momentum

$$(\tau)(t) = I(2\omega) - I\left(\frac{4}{3}\omega\right) = \frac{2I\omega}{3}$$

$$\tau = \frac{2I\omega}{3t}$$

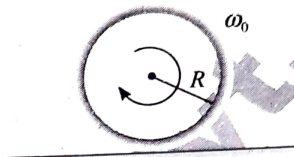
15. (b) $\Delta K = K_i = K_f$

$$= \left[\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2 \right] - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2 = \frac{1}{3}I\omega^2$$

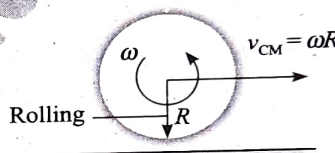
DPP 3.3

Subjective Type

1. (a) Before being brought in contact with the table the disc was in pure rotational motion hence, $v_{CM} = 0$.

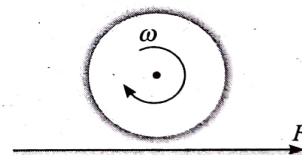


- (b) When the disc is placed in contact with the table due to friction velocity of a point on the rim decreases.
 (c) When the rotating disc is placed in contact with the table due to friction centre of mass acquires some linear velocity.
 (d) Friction is responsible for the effects in (b) and (c).
 (e) When rolling starts $v_{CM} = \omega R$
 where ω is angular speed of the disc when rolling just starts.



- (f) Acceleration produced in centre of mass due to friction

$$a_{CM} = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g$$



Angular retardation produced by the torque due to friction.

$$\alpha = \frac{\tau}{I} = \frac{\mu_k mgR}{I}$$

$$[\because \tau = (\mu_k N)R = \mu_k mgR]$$

$$\therefore v_{CM} = u_{CM} + a_{CM}t$$

$$\Rightarrow v_{CM} = \mu_k gt$$

$$(\because u_{CM} = 0)$$

$$\text{and } \omega = \omega_0 + \alpha t \Rightarrow \omega = \omega_0 - \frac{\mu_k mgR}{I}t$$

For rolling without slipping,

$$\frac{v_{CM}}{R} = \omega$$

$$\Rightarrow \frac{v_{CM}}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

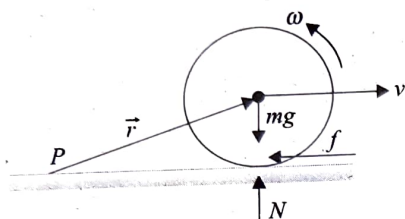
$$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k mgR}{I}t$$

$$t = \frac{R\omega_0}{\mu_k g \left(1 + \frac{mR^2}{I} \right)}$$

S.24 Mechanics II

2. If we choose the point P , friction f cannot produce a torque as it passes through P ; $m\vec{g}$ and $\vec{N}$ being collinear, equal and opposite, torques produced by them nullify each other. Then, no net torque is experienced by the rolling body. Hence, we can conserve the angular momentum of the body about P .

As we know, $L_P = \vec{L} = \vec{r} \times m\vec{v} + \frac{2}{5}mR^2\vec{\omega}$



$$= -mvR\hat{k} + \frac{2}{5}mR^2\omega\hat{k}$$

$$= \left(\frac{2}{5}mR^2\omega - mvR\right)\hat{k}$$

The body will stop spinning and sliding simultaneously, when $v = 0$ and $\omega = 0$

Hence $\vec{L}_2 = \vec{L}_1 = 0$

Substituting $\omega = \omega_0$ and $v = v_0$

We have $\vec{L}_2 = \vec{L}_1 (= \vec{L}_0) = \left(\frac{2}{5}mR^2\omega_0 - mv_0R\right)\hat{k} = 0$

This gives $\omega_0 = \frac{5v_0}{2R}$

Single Correct Answer Type

3. (b) For a ball rolling without slipping on a fixed rough surface, no work is done by friction.
4. (c) Since angular velocity is constant, acceleration of centre of mass of disc is zero. Hence the magnitude of acceleration of point S is $\omega^2 x$ where ω is the angular speed of disc and x is the distance of S from centre. Therefore the graph is a straight line
5. (a) Linear momentum is conserved
 $mv = mv'$ or $v = v'$
 Angular momentum about centre of disc

$$= mvh = mv \cdot \frac{r}{2} = I\omega$$

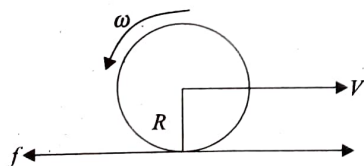
$$\text{So, } mv \cdot \frac{r}{2} = I\omega = \frac{mr^2}{2} \cdot \omega$$

$$\text{or } v = r\omega$$

Pure rolling starts from beginning.

6. (b) $a = f/m = \mu g$

$$\alpha = \tau/I = \frac{(\mu mg)R}{(mR^2/2)} = \frac{2\mu g}{R}$$



$$V = V_0 + at_1$$

$$t_1 = \frac{V}{a} = \frac{V}{\mu g}$$

$$\omega = \omega_0 + \alpha t_2 = t_2 = \frac{\omega}{\alpha} = \frac{\omega R}{2\mu g}$$

Cylinder stops at $t_1 = t_2$

Which gives $R = 2V/\omega$

7. (d) Angular momentum about A is conserved.

$$L_1 = \frac{2}{5}mR^2\omega_0$$

$$L_2 = \frac{2}{5}mR^2\omega + mR^2\omega = \frac{7}{5}mR^2\omega$$

$$L_1 = L_2$$

$$\omega = \frac{2}{7}\omega_0$$

$\omega_0 > \omega$
 and $V > V_0$
 Sphere accelerates forward and rotation decelerates.

$$\omega = \omega_0 - \alpha t$$

$$\alpha = \frac{\omega_0 - \omega}{t}$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$\theta = \frac{(\omega_0 - \omega)(\omega + \omega_0)}{2\alpha} = \frac{\alpha t \cdot \left(\omega + \frac{7}{2}\omega\right)}{2\alpha} = \frac{9}{4}\omega t$$

$$R\theta = \frac{9}{4}\omega R t = \frac{9}{4}\omega t$$

$$V = V_0 + at \quad (V_0 = 0)$$

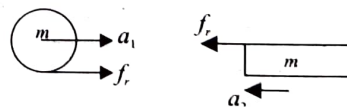
$$a = \omega R/t$$

$$V^2 + V_0^2 + 2aS$$

$$S = \frac{V \cdot V}{2a} = \frac{(\omega R)(V \cdot t)}{2 \cdot \omega R} = \frac{Vt}{2}$$

Ratio of distance = 9/2

8. (d) FBD for sphere and block



$$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m} \quad a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$$

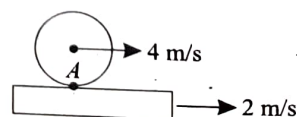
$$\vec{a}_1 = \mu g \hat{i}$$

$$\vec{a}_2 = -\mu g \hat{i}$$

$$\vec{a}_{\text{rel}} = \vec{a}_1 - \vec{a}_2 = 2\mu g \hat{i}$$

$$a_{\text{rel}} = 2\mu g$$

9. (a) For rolling, $v_A = 2 \text{ m/s}$



$$\text{or } 4 - 1 \cdot \omega = 2$$

$$\text{or } \omega = 2 \text{ rad/sec (clockwise)}$$

(d) For a rigid body in uniform pure rolling, there is no friction, hence irrespective of the nature of surface on which it rolls (as long as the surface is rigid), it will continue in its state of pure rolling with uniform velocity.

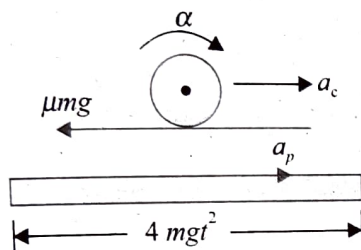
1. (c) Since the spool does not slide, the torque on spool about its point of contact on table is only due to applied force. F_3 causes spool to rotate anticlockwise (about point O), therefore centre of mass should shift to left.



F_2 causes no torque about point O , hence there is no shift of centre of mass under given conditions. F_1 causes spool to rotate clockwise (about point O) and hence centre of mass shifts to right.

12. (b) Acceleration of cylinder

$$a_c = -\mu g$$



Acceleration of plank $= a_p = \mu g$

$$\alpha_c = \frac{\mu mg R}{(mR^2/2)} = \frac{2\mu g}{R}$$

For pure rolling after t ,

$$V_p = V_c - R\omega_c$$

$$a_p t = (V_0 - a_c t) - R\alpha_c t$$

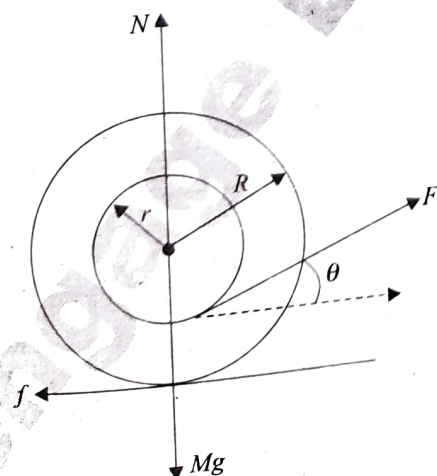
$$\text{or } \mu g t = (V_0 - \mu g t) - 2\mu g t$$

$$t = V_0 / 4\mu g$$

Comprehension Type

13. (a) $F - f = ma$

$$F(r) + F(R) = I\alpha$$



$$a = R\alpha$$

$$\text{Solving, } a = \frac{F(r+R)R}{(I+mR^2)} \text{ and } f = F - ma$$

$$\text{We get } = \frac{F[(I/m) - Rr]}{[(I/m) + r^2]}$$

14. (a) f is positive for $(I/m) > Rr$ or frictional force acts in backward direction.

15. (a) f is negative for $(I/m) < Rr$ or frictional force acts in forward direction.

DPP 3.4

Subjective Type

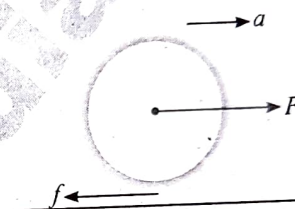
1. Consider the diagram below

Frictional force (f) is acting on the opposite direction of F .

Let the acceleration of centre of mass of disc be a , then

$$F - f = Ma$$

where M is mass of the disc



The angular acceleration of the disc is

$$\alpha = a/R \quad (\text{for pure rolling})$$

From $\tau = I\alpha$

$$\Rightarrow fR = \left(\frac{1}{2}MR^2\right)\alpha$$

$$\Rightarrow fR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$$\Rightarrow Ma = 2f$$

From Eqs. (i) and (ii) we get

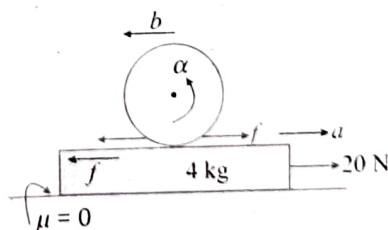
$$f = F/3 \quad [\because N = Mg]$$

$$\therefore f \leq \mu N = \mu Mg$$

$$\Rightarrow \frac{F}{3} \leq \mu Mg \Rightarrow F \leq 3\mu Mg$$

$$\Rightarrow F_{\max} = 3\mu Mg$$

2. Suppose acceleration of cylinder w.r.t. plank is ' b ' w.r.t. plank.



As there's no slipping $b = R\alpha$

Equation of rotation motion $fR = I\alpha$

$$\Rightarrow f = \frac{M_c R^2}{2 \cdot R} \cdot \frac{b}{R} = \frac{M_c b}{2}$$

Linear motion of cylinder

$$f = M_c(a - b)$$

For plank

$$20 - f = 4a$$

Putting $M_c = 2 \text{ kg}$

$$f = b \quad 2f = 2(a - b)$$

$$\Rightarrow f = \frac{2a}{3}$$

$$\text{Using in (4)} \quad 20 - \frac{2a}{3} = 4a \Rightarrow a = \frac{30}{7} \text{ m/s}^2$$

$$\text{and } b = \frac{20}{7} \text{ m/s}^2$$

$$\text{Acceleration of cylinder } a = \frac{10}{7} \text{ m/s}^2$$

$$3. \quad mg \sin \theta - f = ma$$

$$f \times r = I \times \frac{a}{r} \therefore mg \sin \theta - \frac{Ia}{r^2} = ma$$

$$\therefore a = \frac{mg \sin \theta}{m + \frac{I}{r^2}}$$

$$I_A = mr^2 \text{ and } I_B = \frac{1}{2}mr^2 \therefore I_A > I_B \therefore a_A < a_B$$

So 'B' will reach first at bottom.

So by second equation of motion for 'B'

$$l_B = \frac{h}{\sin \theta} = 0 + \frac{1}{2} \times \frac{mg \sin \theta}{m + \frac{1}{2}m} t^2 = \frac{1}{2} \times \frac{2}{3} g \sin \theta t^2$$

$$\Rightarrow t^2 = \frac{3h}{g \sin^2 \theta}$$

$$\text{for A } l_A = 0 + \frac{1}{2} \times \frac{g \sin \theta}{2} t^2 = \frac{g \sin \theta}{4} \times \frac{3h}{g \sin^2 \theta}$$

$$l_A = \frac{3h}{g \sin \theta}$$

So required distances = $l_B - l_A$

$$d = \frac{h}{4 \sin \theta}$$

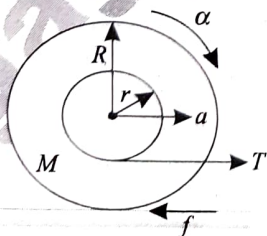
4. Since the spool rolls without slipping $a = R\alpha$
The F.B.D. of spool is shown in the following figure.

For linear motion

$$T - f = Ma$$

and for rotational motion

$$f \cdot R - T \cdot r = \frac{MR^2}{2} \left(\frac{a}{R} \right)$$

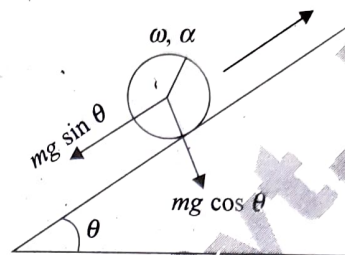
For maximum value of T , $f = \mu Mg$ 

$$\text{On solving } T = \frac{3\mu MgR}{R + 2r}$$

$$\text{Using values } T = \frac{3 \times (0.4) \times 3 \times 10 \times 0.2}{0.2 + 0.2} = 18 \text{ N}$$

Single Correct Answer Type

$$5. (a) \quad \tau = I\alpha = \frac{mr^2}{2} \alpha = \mu mg \cos \theta \cdot r$$



$$\text{or } \alpha = \frac{2\mu g \cos \theta}{r}$$

Retarding force for linear velocity

$$= mg \sin \theta - \mu mg \cos \theta$$

Linear deceleration

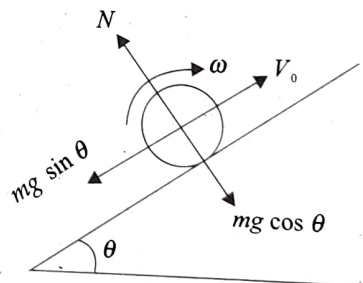
$$= \frac{1}{m} [ma - (mg \sin \theta - \mu mg \cos \theta)]$$

$$r\alpha = \mu g \cos \theta - g \sin \theta$$

$$= 3\mu g \cos \theta - g \sin \theta \quad [\text{from eq. (i)}]$$

$$= g(3\mu \cos \theta - \sin \theta)$$

6. (a) Retarding force for linear velocity = $mg \sin \theta$. All the forces pass through the centre. There is no force to provide moment to reduce angular velocity. So angular velocity will remain constant up to the highest point. The sphere will stop going up at a point where translational velocity becomes zero.



$$\frac{m(V_0 - 0)}{t} = mg \sin \theta \quad \text{or } t = V_0 / g \sin \theta$$

$$7. (a) \quad \omega_0 = \frac{2V_0}{3R} \text{ or } V_0 = \frac{3}{2} R\omega_0$$

Angular momentum is conserved about A.

$$L_1 = L_2$$

$$\frac{2}{5} mR^2 \omega_0 + mV_0 R = \frac{2}{5} mR^2 \omega + mVR$$

$$\text{or } \frac{2}{5} mR^2 \times \frac{2V_0}{3R} + mV_0 R = \frac{2}{5} mR^2 \frac{V}{R} + mVR$$

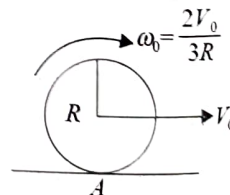
$$\text{or } V_0 = \frac{21}{19} V$$

(i)

 ω and V are the angular and translational velocities when pure rolling starts.

$$\omega = \omega_0 + \alpha t$$

$$\text{From eq. (i)} \quad \alpha = \frac{5V}{19Rt}$$



Where t is the time in which pure rolling starts.
Pure rolling starts when sphere rotates by an angle,

$$\theta = \omega_0 t + \frac{1}{2} \alpha^2 = \frac{2V_0}{3r} t + \frac{1}{2} \cdot \frac{5V}{19Rt} \cdot t^2 = \frac{33}{38} \frac{Vt}{R}$$

$$R\theta = S = \frac{33}{38} Vt$$

$$V = \frac{38}{33} \cdot \frac{S}{t}$$

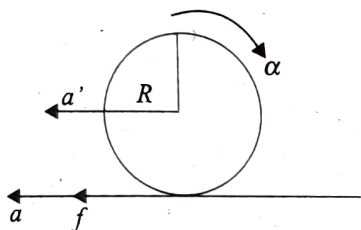
8. (d) $\theta = \frac{33}{38} \frac{Vt}{R}$

$$R\theta = S = \frac{33}{38} Vt$$

$$V = \frac{38}{33} \cdot \frac{S}{t}$$

9. (c) $Mg - T = ma$
and, $T = (2m)a$

From eqs. (i) and (ii) $a = \frac{Mg}{(M+2m)}$



$$a' = f/m$$

$$\alpha = \tau/I = \frac{fR}{(mR^2/2)}$$

For no slipping, $a = a' = \alpha R$

$$\frac{Mg}{(M+2m)} = \frac{f}{m} + \frac{2fR}{MR}$$

Which gives $f = \frac{Mmg}{3(M+2m)}$

10. (a) Initial situation w.r.t. the conveyer belt:



For sphere : initial velocity $(\omega) = V$

$$\mu mg R = \frac{2}{5} m R^2 \alpha \Rightarrow \alpha = \frac{5}{7} \cdot \frac{g}{R}$$

At pure rolling:

$$v' = v + (\mu g)t \text{ and } \omega' = 0 + \left(\frac{5}{7} \cdot \frac{g}{R}\right)t$$

(Since $v' = R\omega'$ at pure rolling)

$$\Rightarrow v - \mu g t = \frac{5}{7} \cdot g \cdot t$$

$$\Rightarrow t = \frac{v}{g} \left(\because \mu = \frac{2}{7} \right)$$

For centre of the sphere: $S = v \left(\frac{v}{g} \right) - \frac{1}{2} \left(\frac{2}{7} \cdot g \right) \left(\frac{v^2}{g^2} \right)$

$$S = \frac{v^2}{g} - \frac{v^2}{7g}$$

(w.r.t. belt)

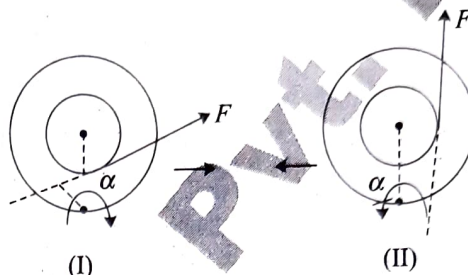
In this time, the belt with speed ' v ' will move a distance of

$$s_b = vt = v \left(\frac{v}{g} \right) = \frac{v^2}{g}$$

$\therefore$ Distance travelled w.r.t. ground will be:

$$S_g = |S - s_b| = \left| \left(\frac{v^2}{g} - \frac{v^2}{7g} \right) - \left(\frac{v^2}{g} \right) \right| = \frac{v^2}{7g}$$

11. (a)



(i)
(ii)

(Pure rotation about instantaneous point of contact)

Note: If line of action passes through point of contact, it only spins.

12. (d) Since linear acceleration is the same for all ($a = Mg \sin \theta - \mu Mg \cos \theta$) as they have the same mass ' M ' and the same ' μ '. Hence, all will reach the bottom simultaneously. Hence (d).

13. (b) For all the bodies, torque is the same.
Hence, angular momentum (L) is also the same.

$$\text{Now, KE} = \frac{1}{2} MV^2 + \frac{L^2}{2I}$$

Linear velocity ' v ' is the same for all as the same force acts on them. Therefore more value of moment of inertia implies lesser kinetic energy.

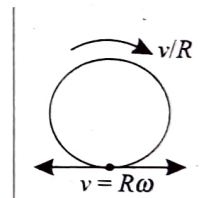
Among all, the hollow sphere has the maximum moment of inertia $I = \left(\frac{2}{3} MR^2 \right)$.

14. (d) As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

$$v_{\text{net}} \text{ (for lowest point)} = v - R\omega = v - v = 0$$

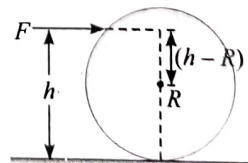
$$\text{and acceleration} = \frac{v^2}{R} + 0 = \frac{v^2}{R}$$

(Since linear speed is constant)
Hence (d).



Matching Column Type

15. Consider the diagram where a sphere of m and radius R struck horizontally at height h above the floor.



The sphere will roll without slipping when $\omega = \frac{v}{r}$, where v is the linear velocity and ω is the angular velocity of the sphere.

Now, angular momentum of sphere is about centre of mass
[We are applying conservation of angular momentum just before and after it is struck]

$$mv(h-R) = I\omega = \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)$$

$$\Rightarrow mv(h-R) = \frac{2}{5}mvR$$

$$h-R = \frac{2}{5}R \Rightarrow h = \frac{7}{5}R$$

Therefore, the sphere will roll without slipping with a constant velocity and, hence, no loss of energy, so (s) $\rightarrow$ (i)

Torque due to applied force, F about centre of mass

$$\tau = F(h-R) \text{ (clockwise)}$$

For, $\tau = 0$, $h = R$ sphere will have only translational motion. It would lose energy by friction.

Hence, (q) $\rightarrow$ (iv)

The sphere will spin clockwise when

$$\tau > 0 \Rightarrow h > R$$

Therefore, (r) $\rightarrow$ (iii)

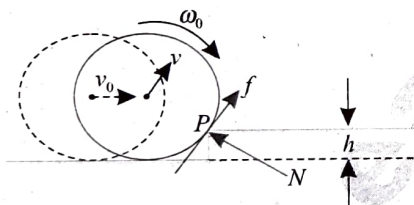
The sphere will spin anticlockwise when

$$\tau < 0 \Rightarrow h < R \quad (p) \rightarrow (iii)$$

DPP 3.5

Subjective Type

1. In the FBD we have shown the impulsive forces, i.e., friction f and normal contact force N acting at the point of impact P . The gravitational force is ignored because it is nonimpulsive.



If we choose the reference point P , as the forces f and N are passing through this point, they cannot produce torques about P . Then the angular impulse of both forces f and N are zero about P . Hence we can conserve the angular momentum of the body about P just before and just after the impact.

$$\bar{L}_{1P} = \bar{L}_{2P}$$

$$\bar{L}_{1P} = mv_0(R-h) \downarrow + \frac{2}{5}mR^2\omega_0 \downarrow$$

$$\bar{L}_{2P} = \left(\frac{2}{5}mv_0 + mR^2\right)\omega \downarrow$$

Because the body rotates perfectly about P just after the collision as it does not slide and bounce.

$$v_0(R-h) + \frac{2}{5}R^2\omega_0 = \frac{7}{5}R^2\omega$$

As the body was rolling before the impact, $v_0 = R\omega_0$

Putting $v_0 = R\omega_0$ in the above expression.

$$\omega = \left(1 - \frac{5h}{7R}\right)\frac{v_0}{R}$$

Single Correct Answer Type

2. (d) Since the two bodies have the same mass and collide head-on elastically, the linear momentum gets interchanged.

Hence just after the collision 'B' will move with velocity ' v_0 ' and 'A' becomes stationary but continues to rotate at the same

initial angular velocity $\left(\frac{v_0}{R}\right)$.

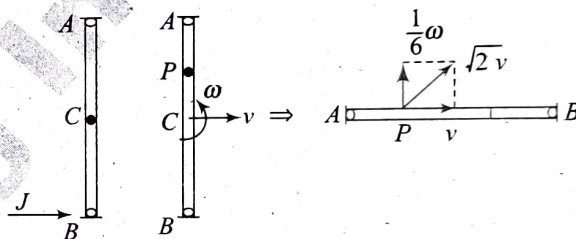
Hence, after collision.

$$(KE)_B = \frac{1}{2}mv_0^2$$

$$\text{and } (KE)_A = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v_0}{R}\right)^2$$

$$\Rightarrow \frac{(K.E.)_B}{(K.E.)_A} = \frac{3}{2} \text{ Hence (d).}$$

3. (d) Let v and ω be the linear and angular speeds of the rod after applying an impulse J at B .



Then from:

Impulse = change in momentum

$$\text{We have } mv = J \text{ or } v = \frac{J}{m} \quad (i)$$

$$I\omega = J \cdot \frac{l}{2}$$

$$\frac{ml^2}{12} \cdot \omega = J \cdot \frac{l}{2} \text{ or } \omega = \frac{6J}{ml} \quad (ii)$$

After the given time, $t = \frac{\pi ml}{12J}$ the rod will rotate an angle

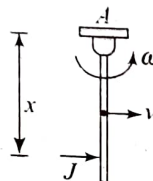
$$\theta = \omega t = \left(\frac{6J}{ml}\right)\left(\frac{\pi ml}{12J}\right) = \frac{\pi}{2}$$

$$\frac{l}{6} \cdot \omega = \left(\frac{l}{6}\right)\left(\frac{6J}{ml}\right) = \frac{J}{m} = v$$

$$|\vec{v}_P| = \sqrt{2}v = \sqrt{2} \frac{J}{m}$$

4. (c) From impulse = change in momentum we have

$$J = mv \quad (i)$$



(v = linear speed of centre of mass of rod)

$$J \cdot x = I\omega = \frac{ml^2}{3}\omega$$

$$v = \frac{l}{2}\omega$$

(ii)

(iii)

(ω = angular speed of rod about point A)

Solving these three equations, we get

$$x = \frac{2}{3}l$$

5. (d) Let after collision velocity of rod be v and angular velocity be ω , then

$$mv_0 = mv + MV \Rightarrow mv_0x = mvx + \frac{M\ell^2}{12}\omega$$

$$mv_0 = mv + \frac{M\omega\eta^2x}{12}$$

From above equations,

$$\frac{M\omega\eta^2x}{12} = MV \Rightarrow \omega\eta^2x = 12V$$

$$\text{Velocity of farther end of the rod} = V - \frac{l\omega}{2}$$

$$= \frac{\omega\eta^2x}{12} - \frac{\omega\eta x}{2} = \frac{\omega\eta x}{2} \left(\frac{\eta}{6} - 1 \right)$$

If this velocity is opposite to v_0 , then

$$\frac{\eta}{6} - 1 < 0 \Rightarrow \eta < 6$$

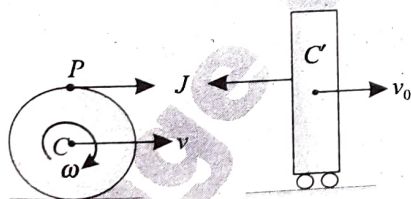
Comprehension Type

6. (b), 7. (d), 8. (a)

Just after the string becomes taut due to the impulse of the tension, that is, $\int Tdt = J$ say, the linear momentum changes from 0 to mv .

$$J = mv$$

$$v = \frac{J}{m}$$



The angular impulse of the tension about $C = JR$, which changes the angular momentum from 0 to $I_c\omega$.

$$\text{Where } I_c = \frac{mR^2}{2}$$

$$\text{This gives } \omega = \frac{2J}{mR}$$

The velocity of P is $\vec{v}_P = \vec{v}_{PC} + \vec{v}_C$

Where $v_{PC} = \omega R$ and $v_C = v$

This gives $v_P = \omega R + v$

Then substituting ω and v , we have

$$v_P = \frac{2J}{m} + \frac{J}{m} = \frac{3J}{m}$$

Since the string is inextensible, $v_P = v_0$

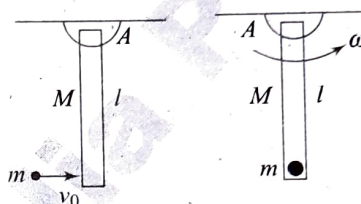
$$\frac{3J}{m} = v_0$$

$$\text{This gives } J = \frac{mv_0}{3}$$

$$\text{Hence } v = \frac{v_0}{3} \text{ and } \omega = \frac{2v_0}{3R}$$

9. (a), 10. (b), 11. (d)

9. (a)



Apply conservation of angular momentum about A:

$$mv_0l = \frac{ml^2}{3}\omega + m(\omega l)l$$

$$\omega = \frac{v_0}{2l}$$

$$10. (b) \text{ Fractional loss: } f = \frac{KE_f}{KE_i}$$

$$= 1 - \left(\frac{\frac{1}{2} \left(\frac{Ml}{3} \right)^2 \omega^2 + \frac{1}{2} m(\omega l)^2}{\frac{1}{2} mv_0^2} \right) = \frac{3}{4}$$

11. (d) Let cm of rod raises height h , then m will raise height $2h$

$$Mgh + mg2h = KE_f$$

$$h = \frac{v_0^2}{20g}$$

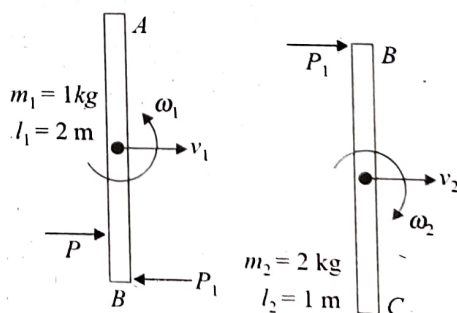
12. (a), 13. (b), 14. (c), 15. (a)

Let P_1 is the impulse at B between the rods.

$$P - P_1 = m_1v_1 \Rightarrow 10 - P_1 = v_1$$

(i)

$$P \times 0.5 - P_1 \times 1 = \frac{m_1 l_1^2}{12} \omega_1$$



$$5 - P_1 = \frac{\omega_1}{3}$$

(ii)

S.30 Mechanics II

$$P_1 = m_2 v_2 = 2v_2$$

(iii)

$$P_1 \times 0.5 = \frac{m_2 l_2^2}{12} \omega_2$$

$$\frac{P_1}{2} = \frac{\omega_2}{6}$$

(iv)

Constraint relation: velocity of end B is the same for both rods.

$$v_1 + \omega_1 \times 1 = v_2 + \omega_2 \times 0.5$$

(v)

Now put from first four equations into (v)

$$10 - P_1 + 3[5 - P_1] = \frac{P_1}{2} + \frac{6P_1}{2} \times 0.5 \Rightarrow P_1 = \frac{25}{6} \text{ Ns}$$

$$\omega_2 = 3P_1 = \frac{3 \times 25}{6} = 12.5 \text{ rad/s}$$

$$v_1 = 10 - P_1 = 10 - \frac{25}{6} = \frac{35}{6} \text{ m/s}$$

$$L_{BC} = \frac{m_2 l_2^2}{12} \omega_2 = \frac{2 \times 1^2}{12} \times \frac{25}{2} = \frac{25}{12} \text{ kg m}^2/\text{s}$$

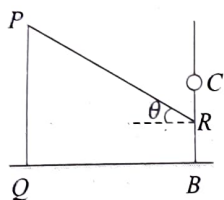
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CHAPTER 4

DPP 4.1

Subjective Type

1. $\tan \theta = a/g = 4/5$
 $\Rightarrow RB = PQ - QB \tan \theta = 2 - 2 \times 4/5 = 0.4 \text{ m}$

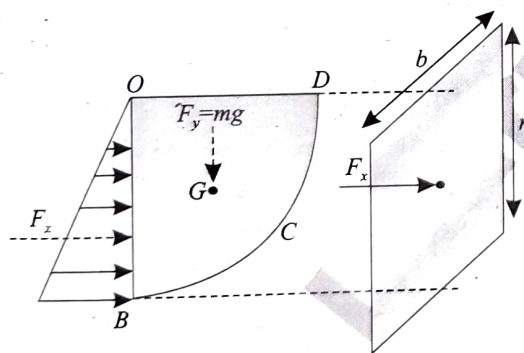


$$\text{Volume of water contained} = \left(\frac{PQ + BR}{2} \right) \times 4 \text{ m}^2$$

$$\text{Initial volume of water} = BD \times 4 \text{ m}^2$$

$$\begin{aligned} \text{Amount of spill} &= V_{\text{in}} - V_{\text{fin}} \\ &= 2(2BD - PQ - BR) \times 1000 \text{ litres} \\ &= 2(3 - 2 - 0.4) \times 1000 \\ &= 1200 \text{ litres} \end{aligned}$$

2. Figure shows the free body diagram of the volume of water enclosed in BDO.



The horizontal component of force equal to the hydrostatic force on the projection of dam on vertical plane.

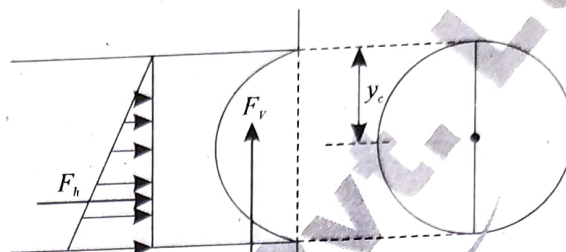
$$\begin{aligned} F_x &= \rho g h A = \frac{\rho g r}{2} b r \\ &= \frac{1000 \times 10 \times 4 \times 30 \times 4}{2} = 2400 \text{ kN} \end{aligned}$$

The weight of water acts at the center of mass G of the quarter circular section.

$$mg = \rho g V = (1000)(10) \frac{\pi(4)^2}{4} (30) = 1200\pi \text{ kN}$$

Hence vertical component of hydrostatic force $F_y = mg = 1200\pi \text{ kN}$

3. (a) Let the horizontal and vertical thrusts on the hemisphere be F_h and F_v respectively.



Projection of gate on vertical plane

We know that $F_h = \rho g y_c A_v$

Where $y_c = R$

And $A_v = \pi R^2$

This gives $F_h = \rho g \pi R^3$ (right)

- (b) The vertical thrust on the hemisphere will be equal to weight of the water displaced by it, i.e., buoyancy force.

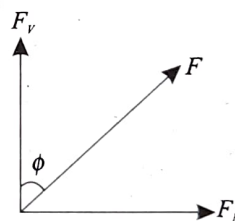
Hence $F_v = \rho V g$

Where V = volume of the hemisphere $= \frac{2}{3} \pi R^3$

We have $F_v = \frac{2}{3} \rho g \pi R^3$ (up)

- (c) Hence the net hydrostatic force on the hemisphere is

$$F = \sqrt{F_h^2 + F_v^2}$$



$$F = \sqrt{(\rho g \pi R^3)^2 + \left(\frac{2}{3} \rho g \pi R^3 \right)^2} = \frac{\sqrt{13}}{3} \rho g \pi R^3$$

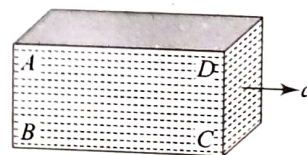
- (d) The angle of orientation of the force F with vertical

$$\begin{aligned} \phi &= \tan^{-1} \frac{F_h}{F_v} \\ &= \tan^{-1} \frac{\rho g \pi R^3}{\frac{2}{3} \rho g \pi R^3} = \tan^{-1} \frac{3}{2} \end{aligned}$$

Single Correct Answer Type

4. (b) As long as $\rho \leq \rho_w$, pressure at the bottom of the pan would be the same everywhere, according to the Pascal's law.

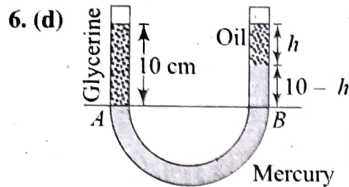
5. (a) Due to acceleration towards right, there will be a pseudo force in a left direction. So the pressure will be more on rear side (Points A and B) in comparison with front side (Point D and C).



S.32 Mechanics II

Also due to height of liquid column pressure will be more at the bottom (points B and C) in comparison with top (point A and D).

So overall maximum pressure will be at point B and minimum pressure will be at point D.



At the condition of equilibrium

Pressure at point A = Pressure at point B

$$P_A = P_B \Rightarrow 10 \times 1.3 \times g = h \times 0.8 \times g + (10 - h) \times 13.6 \times g$$

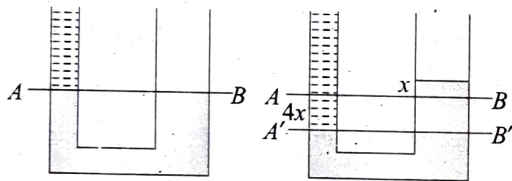
By solving, we get $h = 9.7$ cm

7. (b) Thrust on lamina = pressure at centroid $\times$ Area

$$= \frac{h\rho g}{3} \times A = \frac{1}{3} A\rho gh$$

8. (c) If the rise of level in the right limb be x cm, the fall of level of mercury in left limb be $4x$ cm because the area of cross section of right limb is 4 times as that of left limb.

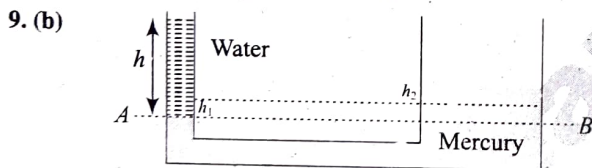
$\therefore$ Level of water in left limb is $(36 + 4x)$ cm.



Now equating pressure at interface of Hg and water (at $A'B'$)

$$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$$

By solving we get $x = 0.56$ cm



If the level in narrow tube goes down by h_1 , then in wider tube goes up to h_2

$$\text{Now, } \pi^2 h_1 = \pi(nr)^2 h_2 \Rightarrow h_1 = n^2 h_2$$

Now, pressure at point A = pressure at point B

$$h\rho g = (h_1 + h_2)\rho'g$$

$$\Rightarrow h = (n^2 h_2 + h_2)sg$$

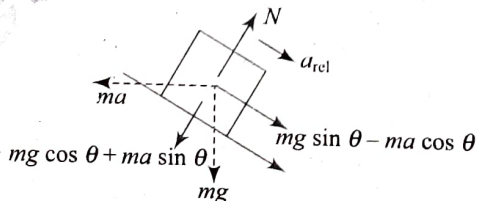
$$\left(\text{As } s = \frac{\rho'}{\rho} \right)$$

$$h_2 = \frac{h}{(n^2 + 1)s}$$

10. (b) $\tan \theta = \frac{a}{g} = \frac{h_2 - h_1}{h_2 \tan 45^\circ + h_1 \tan 45^\circ} = \frac{4 \text{ cm}}{20 \text{ cm}}$

$$\Rightarrow a = 2 \text{ m/s}^2$$

11. (a)



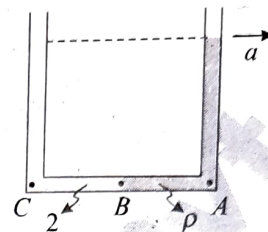
$$ma_{\text{rel}} = mg \sin \theta - ma \cos \theta$$

But for water surface $\tan \theta = a/g$

$$\Rightarrow a_{\text{rel}} = 0$$

12. (b) For the given situation, liquid of density 2ρ should be behind that of ρ .

From right limb:



$$P_A = P_{\text{atm}} + \rho gh$$

$$P_B = P_A + \rho a \frac{l}{2} = P_{\text{atm}} + \rho gh + \rho a \frac{l}{2}$$

$$P_C = P_B + (2\rho)a \frac{l}{2} = P_{\text{atm}} + \rho gh + \frac{3}{2} \rho al \quad (1)$$

$$\text{But from left limb } P_C = P_{\text{atm}} + (2\rho)gh \quad (2)$$

From (1) and (2);

$$P_{\text{atm}} + \rho gh + \frac{3}{2} \rho al = P_{\text{atm}} + 2\rho gh$$

$$\Rightarrow h = \frac{3a}{2g}l$$

13. (a) The force exerted on the spring, via the piston by water = (pressure at centroid of the piston) $\times$ (area of the piston)

$$= \left(\rho g \frac{h}{2} \right) (ah)$$

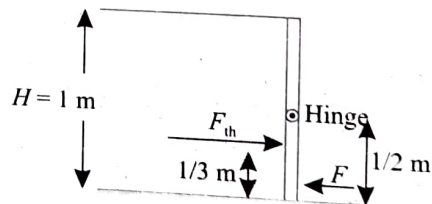
$$F = 1 \times 1000 \times \frac{20^2}{2} \times 10 = 2 \times 10^5 \text{ dyne}$$

Now, if x cm be the compression of the spring, then for equilibrium $F = Kx$

$$2 \times 10^6 = 1500 \times 10^3 \frac{\text{dyne}}{\text{cm}} \times x \text{ cm}$$

$$x = 20/15 = 1.3 \text{ cm}$$

14. (c)



$$F_{\text{th}} = \frac{1}{2} \rho g H^2 \cdot b = \frac{1}{2} \times 10^3 \times 10 \times 1^2 \times 3$$

$$F_{\text{th}} = \frac{3}{2} \times 10^4 \text{ N}$$

Taking torque about the hinge,

$$F_{\text{th}} \times \left(\frac{1}{2} - \frac{1}{3} \right) = F \cdot \frac{1}{2}$$

$$F_{\text{th}} \cdot \frac{1}{6} = F \cdot \frac{1}{2}$$

$$F = \frac{1}{3} \cdot F_{\text{th}} = \frac{1}{2} \times 10^4 \text{ N}$$

Multiple Correct Answers Type

15. (b, c)

Pressure is defined as the ratio of magnitude of component of the force normal to the area and the area under consideration. As magnitude of component is considered, hence, it will not have any direction. So, pressure is a scalar quantity.

DPP 4.2

Single Correct Answer Type

1. (b) Mass of water in the beaker = $250 \times 1 = 250$ g

Mass of beaker + Mass of water in it
= mass of the volume of water displaced
= $400 + 250 = 650$ g
So volume of water displaced
= 650 cm^3

As the inner volume = 500 cm^3
So volume of the material of beaker
= $650 - 500 = 150 \text{ cm}^3$

$$\therefore \text{Density} = \frac{400}{150} = 2.67 \text{ g/cm}^3$$

2. (c) Apparent weight = $V(\rho - \sigma)g = \frac{m}{\rho}(\rho - \sigma)g$

where m = mass of the body,
 ρ = density of the body
 σ = density of water

If two bodies are in equilibrium, then their apparent weight must be equal.

$$\therefore \frac{m_1}{\rho_1}(\rho_1 - \sigma) = \frac{m_2}{\rho_2}(\rho_2 - \sigma)$$

$$\Rightarrow \frac{36}{9}(9 - 1) = \frac{48}{\rho_2}(\rho_2 - 1)$$

By solving we get $\rho_2 = 3$

3. (d) Let M_0 = mass of body in vacuum
Apparent weight of the body in air = Apparent weight of standard weights in air

$\Rightarrow$ Actual weight - upthrust due to displaced air
= Actual weight - upthrust due to displaced air

$$\Rightarrow M_0 g - \left(\frac{M_0}{d_1}\right) dg = Mg - \left(\frac{M}{d_2}\right) dg$$

$$\Rightarrow M_0 = \frac{M \left[1 - \frac{d}{d_2}\right]}{\left[1 - \frac{d}{d_1}\right]}$$

4. (a) Volume of log of wood $V = \frac{\text{mass}}{\text{density}} = \frac{120}{600} = 0.2 \text{ m}^3$

Let x be weight that can be put on the log of wood
So weight of the body = $(120 + x) \times 10 \text{ N}$

Weight of displaced liquid
= $V\sigma g = 0.2 \times 10^3 \times 10 \text{ N}$

The body will just sink in liquid if the weight of the body will be equal to the weight of displaced liquid.

$$\therefore (120 + x) \times 10 = 0.2 \times 10^3 \times 10$$

$$\Rightarrow 120 + x = 200 \therefore x = 80 \text{ kg}$$

5. (c) Weight of the bowl = mg

$$= V\rho g = \frac{4}{3}\pi \left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3 \right] \rho g$$

where D = outer diameter, d = inner diameter and ρ = density of bowl

Weight of the liquid displaced by the bowl

$$= V\sigma g = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 \sigma g$$

where σ is the density of the liquid.

For the floatation,

$$\frac{4}{3}\pi \left(\frac{D}{2}\right)^3 \sigma g = \frac{4}{3}\pi \left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3 \right] \rho g$$

$$\Rightarrow \left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3 \right] 2 \times 10^4$$

By solving we get $d = 0.98 \text{ m}$

6. (b) Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation,

weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g$$

$$\Rightarrow R^3 \rho_1 - r^3 \rho_1 + r^3 \rho_2 = R^3$$

$$\Rightarrow R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \Rightarrow \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1}$$

$$\Rightarrow \frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$$

$$\Rightarrow \frac{(R^3 - r^3)\rho_1}{r^3 \rho_2} = \left(\frac{1 - \rho_2}{\rho_1 - 1}\right) \rho_1$$

$$\Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1 - 0.3}{2.4 - 1}\right) \times \frac{2.4}{0.3} = 4$$

7. (d) Tension in spring T = upthrust - weight of sphere

$$= V\sigma g - V\rho g = V\eta\rho g - V\rho g$$

$$= (\eta - 1)V\rho g = (\eta - 1)mg$$

(As $\sigma = \eta\rho$)

8. (a) When body (sphere) is half immersed, then

upthrust = weight of sphere

$$\Rightarrow \frac{V}{2} \times \rho_{\text{liq}} \times g = V \times \rho \times g \therefore \rho = \frac{\rho_{\text{liq}}}{2}$$

When body (sphere) is fully immersed, then

Upthrust = wt. of sphere + wt. of water poured in sphere

$$\Rightarrow V \times \rho_{\text{liq}} \times g = V \times \rho \times g + V' \times \rho_{\text{liq}} \times g$$

$$\Rightarrow V \times \rho_{\text{liq}} = \frac{V \times \rho_{\text{liq}}}{2} + V' \times \rho_{\text{liq}} \Rightarrow V' = \frac{V}{2}$$

9. (b) The velocity of ball before entering the water surface

$$v = \sqrt{2gh} = \sqrt{2 \times 9 \times 9}$$

When ball enters into water, due to upthrust of water the velocity of ball decreases (or retarded)

$$\text{The retardation, } a = \frac{\text{apparent weight}}{\text{mass of ball}}$$

$$= \frac{V(\rho - \sigma)g}{V\rho} = \left(\frac{\rho - \sigma}{\rho} \right) g = \left(\frac{0.4 - 1}{0.4} \right) \times g = -\frac{3}{2}g$$

If h be the depth up to which ball sink, then

$$0 - v^2 = 2 \times \left(-\frac{3}{2}g \right) \times h \Rightarrow 2g \times 9 \therefore h = 6 \text{ cm}$$

10. (c) Let A = The area of cross section of the hole

v = Initial velocity of efflux

d = Density of water

Initial volume of water flowing out per second = Av

Initial mass of water flowing out per second = Adv

Rate of change of momentum = Adv^2

Initial downward force on the flowing out water = Adv^2

So equal amount of reaction acts upwards on the cylinder.

$\therefore$ Initial upward reaction = Adv^2 [As $v = \sqrt{2gh}$]

$\therefore$ Initial decrease in weight = $Ad(2gh)$

$$= 2Adgh = 2 \times \left(\frac{1}{4} \right) \times 1 \times 980 \times 25 = 12.5 \text{ gm-wt}$$

11. (a) Given that density of ice = $\frac{9}{10}$ × density of water, so only

$\frac{1}{10}$ th of ice will be outside the water. This means that $20 \times \frac{1}{10}$

= 2 m of ice layer will be above the water surface. In other

words, in the hole, the water level will rise up to 18 m of the

ice layer. So minimum length of rope required = 2 m.

12. (a) Let $L = PQ$ = length of rod

$$\therefore SP = SQ = \frac{L}{2}$$

Weight of rod, $W = AL\rho g$, acting

At point S

And force of buoyancy,

$$F_B = AL\rho_0 g, [l = PR]$$

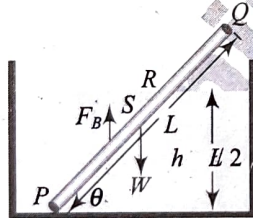
which acts at mid-point of PR .

For rotational equilibrium,

$$AL\rho_0 g \times \frac{l}{2} \cos \theta = AL\rho g \times \frac{L}{2} \cos \theta$$

$$\Rightarrow \frac{l^2}{L^2} = \frac{\rho}{\rho_0} \Rightarrow \frac{l}{L} = \sqrt{\frac{\rho}{\rho_0}}$$

$$\text{From figure, } \sin \theta = \frac{h}{l} = \frac{L}{2l} = \frac{1}{2} \sqrt{\frac{\rho_0}{\rho}}$$



Multiple Correct Answers Type

13. (b, c)

Reading of balance A = weight in air – upthrust

So balance A will read less 2 kg

Liquid exerts an upthrust force U on the body; so body also exerts an equal reactionary force on the liquid. Hence, reading of balance B will increase by an amount to U , i.e., reading of balance $B > 5$ kg.

So options (b) and (c) are correct.

14. (b, c)

Effective value of downward force acting on the body

$$F = V(d - d_0)g$$

So increase in potential energy in lifting the body through height h in the liquid

= work done against effective downward force

$$= V(d - d_0)gh$$

It also evident that potential of water will remain the same by raising the body in it. So options (b) and (c) are correct.

Fill in the Blanks Type

15. Volume of ball = $\frac{40}{0.8} = 50 \text{ cc}$

$$\text{Upthrust} = 50 \times 1 \text{ g}$$

Same will be downward reaction. So net weight

$$= 600 + 50 = 650 \text{ g}$$

16. Let mass of the ball = M grams

$\therefore$ Volume of water displaced, $V = M \text{ cm}^3$

Actual volume sphere

$$v = \frac{M}{2}$$

($\because RD = 2$)

$$\text{So volume of cavity} = V - v = M - \frac{M}{2} = \frac{M}{2} = \frac{V}{2}$$

17. Let l = length of cube outside liquid,

$$\text{Then } (L - l)L^3 \cdot 3\rho = L^3 \cdot \rho \text{ or } l = \frac{2L}{3}$$

18. Let volume of the ball be V

Upthrust on the ball = $V\sigma g$

Net upward force = $V\sigma g - V\rho g$

$$\text{So upward acceleration, } a = \frac{V(\sigma - \rho)g}{V\rho}$$

Velocity on reaching the surface $v = \sqrt{2gH}$

$$\text{Also } v = \sqrt{2gh}$$

$$\therefore 2aH = 2gh$$

$$\text{or } h = \frac{a}{g}H = \left(\frac{\sigma - \rho}{\rho} \right)H$$

19. Upthrust = weight of body

$$\text{For A, } \frac{V_A}{2} \times \rho_W \times g = V_A \times \rho_A \times g \Rightarrow \rho_A = \frac{\rho_W}{2}$$

$$\text{For B, } \frac{3}{4}V_B \times \rho_W \times g = V_B \times \rho_B \times g \Rightarrow \rho_B = \frac{3}{4}\rho_W$$

(Since 1/4 of volume of B is above the water surface)

$$\therefore \frac{\rho_A}{\rho_B} = \frac{\rho_W/2}{3/4\rho_W} = \frac{2}{3}$$

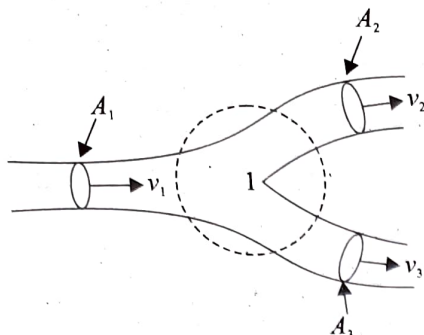
DPP 4.3

Subjective Type

1. If you look at the junction 1, there is no source and no sink
Then, the net incoming flux (rate) = the net outgoing flux.

$$R_1 = R_2 + R_3$$

$$\text{Where } R_1 = A_1 v_1, R_2 = A_2 v_2 \text{ and } R_3 = A_3 v_3$$

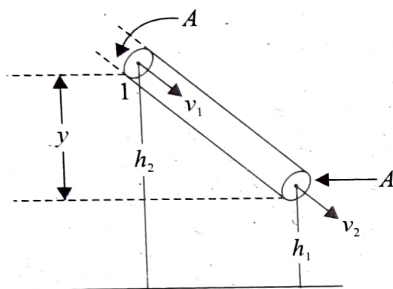


Then $A_1 v_1 = A_2 v_2 + A_3 v_3$

$$v_3 = \frac{A_1 v_1 - A_2 v_2}{A_3} = \frac{(0.01)(2) - (0.02)(3)}{(0.03)} \text{ m/s} = -\frac{4}{3} \text{ m/s}$$

Negative sign signifies that the direction of flow of liquid is just opposite to the assumed direction of v_3 .

2. Since the cross-section of the tube is uniform, $A_1 = A_2 = A$



Then, applying equation of continuity at points 1 and 2, we have

$$v_1 = v_2 \quad (i)$$

Substituting $v_1 = v_2 (= v)$ in Bernoulli's equation

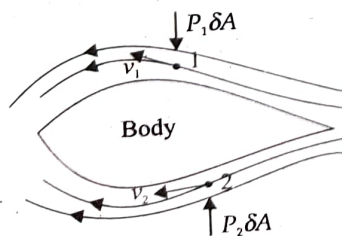
$$\text{We have } P_1 + \frac{1}{2} \rho v^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v^2 + \rho g h_2$$

$$\text{This gives } P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$\text{Since } h_1 - h_2 = h$$

$$\text{We have } P_2 = P_1 + \rho g h$$

3. Let us place a body of any arbitrary shape and size in a flow of liquid (or air or any gas). The body divides the streamlines into two parts as shown in the figure. One part of the streamline moves below and the other part moves above the body. Let us assume that the fluid moves with different speeds v_1 and v_2 at points 1 and 2 of the body. How and why should the fluid move with different speeds at different points of the body is a different question. It depends on shape, size and spin of the body. Furthermore, the texture of the surface also plays a significant role to make the speed of the fluid different at different points of contact of the body. We will discuss some of the above factors in later examples. Here, we emphasize on the principle of "dynamic lift":



As you know $P + \frac{\rho v^2}{2} = c$ when we apply Bernoulli's equation between the two points in a tube of flux at a small vertical distance of separation. Since the distance between points 1 and 2 is negligible compared to the height of the centre of mass of the body, we can write

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2}$$

If $v_1 > v_2$, we have $P_2 > P_1$. Then, the pressure difference is

$$P_2 - P_1 = \frac{\rho}{2} (v_1^2 - v_2^2) \quad (i)$$

This pushes the body from higher pressure to lower pressure with a force

$$F = (P_2 - P_1) A \quad (ii)$$

Where A = effective area

Substituting $P_2 - P_1 = \frac{\rho}{2} (v_1^2 - v_2^2)$ from equation (i) in equation (ii)

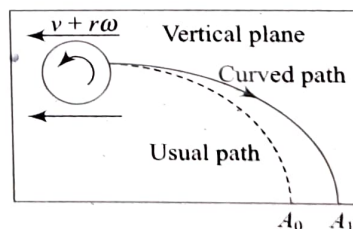
$$\text{We have } F = \frac{\rho A}{2} (v_1^2 - v_2^2)$$

If we put $\frac{v_1 + v_2}{2} = u$ (average velocity of fluid) and $v_1 - v_2 (= u)$

that is the difference in velocities at the points 1 and 2, we have $F = \rho A u v$. Since, this force comes from the motion of the fluid, it is called hydro (or aero)-dynamic force.

Single Correct Answer Type

4. (d) In a streamline flow at any given point, the velocity of each passing fluid particles remains constant. If we consider a cross-sectional area, then a point on the area cannot have different velocities at the same time, hence two streamlines of flow cannot cross each other.
5. (b) As we know for a streamline flow of a liquid velocity of each particle at a particular cross-section is constant, because $Av = \text{constant}$ (law of continuity) between two cross-section of a tube of flow.
6. (c) A stream lined body has less resistance due to air.
7. (b) If a ball is moving from left to right and also spinning about a horizontal axis in anticlockwise direction about a horizontal axis perpendicular to the direction of motion, then relative to the ball air will be moving from right to left.



The resultant velocity of air above the ball will be $(v + r\omega)$ while below it $(v - r\omega)$. So in accordance with Bernoulli's principle, pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following path OA_1 , i.e., if a ball is thrown with back spin the pitch will curve less sharply prolonging the flight.

8. (d) As cross-section areas of both the tubes A and C are the same and tube is horizontal. Hence according to equation of continuity $v_A = v_C$ and therefore according to Bernoulli's theorem $P_A = P_C$ i.e., height of liquid is the same in both the tubes A and C.
9. (b) Bernoulli's theorem for unit mass of liquid

$$\frac{P}{\rho} + \frac{1}{2}v^2 = \text{constant}$$

As the liquid starts flowing, its pressure energy decreases

$$\frac{1}{2}v^2 = \frac{P_1 - P_2}{\rho} \Rightarrow \frac{1}{2}v^2 = \frac{3.5 \times 10^5 - 3 \times 10^5}{10^3} \Rightarrow v^2 = \frac{2 \times 0.5 \times 10^5}{10^3} \Rightarrow v^2 = 100 \Rightarrow v = 10 \text{ m/s}$$

10. (a) When cross-section of duct is decreased, the velocity of water increased and in accordance with Bernoulli's theorem, the pressure P is decreased at that place.

11. (c) Applying continuity equation:

$$V_1 A_1 = V_2 A_2$$

$$A_1 = A_2 \text{ so } V_1 = V_2$$

Applying Bernoulli's equation:

$$P_1 + \rho gh_1 + 1/2 \rho v_1^2 = P_2 + \rho gh^2 + 1/2 \rho v_2^2$$

since $v_1 = v_2$; and $h_1 = h_2$

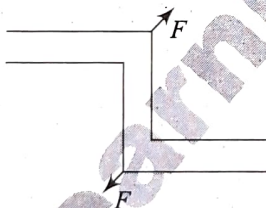
$$\Rightarrow P_1 = P_2$$

12. (b) According to Bernoulli's theorem, $h = \frac{v^2}{2g}$

$$\Rightarrow h = \frac{(2.45)^2}{2 \times 10} = 0.314 = 31.4 \text{ cm}$$

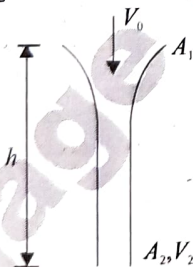
$\therefore$ Height of jet coming from orifice = $31.4 - 10.6 = 20.8 \text{ cm}$

13. (c) The force exerted by the liquid at the bends is shown. The liquid undergoes change in momentum at these points. The two forces from a couple exerting a clockwise torque.



14. (b) $V_2^2 = V_0^2 + 2gh$

$$\text{and } A_1 V_0 = A_2 V_2$$



$$\text{Solving, } \frac{A_2}{A_1} = \frac{V_0}{\sqrt{V_0^2 + 2gh}}$$

$$\frac{A_2}{A_1} = \frac{1}{2} = \frac{V_0}{\sqrt{V_0^2 + 2gh}}$$

$$4V_0^2 = V_0^2 + 2gh$$

$$h = \frac{3V_0^2}{2g}$$

Multiple Correct Answers Type

15. (a, d)

$$P_A = P + p_0 + \rho gh$$

Apply Bernoulli's theorem at A and C and E

$$(p + p_0) + \rho gh = p_c + \rho \frac{V_c^2}{2} \quad (i)$$

$$= p_0 + \rho \frac{V_E^2}{2} \quad (ii)$$

$$p_c = p_0 + \rho gh_1$$

$$A_c V_c = A_E V_E \quad (iii)$$

$$A_c V_c = \left(\frac{A_c}{2}\right) V_E \quad (iv)$$

From eq. (i),

$$(p_0 + \rho gh_1) + \rho \frac{V_c^2}{2} = p_0 + \rho (2V_c)^2$$

$$\rho gh_1 = -4\rho V_c^2 - \frac{\rho}{2} V_c^2 = -\frac{7}{2} \rho (V_c)^2$$

$$\text{or } h_1 = \frac{7V_c^2}{2g}$$

From eq. (ii),

$$(p + p_0) + \rho gh = (p_0 + \rho gh_1) + \rho \frac{V_c^2}{2}$$

$$p + \rho gh = \rho g \left(\frac{7V_c^2}{2} \right)$$

$$v_c^2 = \frac{(p + \rho gh)}{4\rho}$$

$$\text{Discharge rate} = A_c V_c = A \left(\frac{(p + \rho gh)}{4\rho} \right)^{1/2}$$

DPP 4.4

Single Correct Answer Type

$$1. (c) \quad t = \frac{A}{a} \sqrt{\frac{2}{g}} [\sqrt{H_1} - \sqrt{H_2}]$$

$$\text{Now, } T_1 = \frac{A}{a} \sqrt{\frac{2}{g}} \left[\sqrt{H} - \sqrt{\frac{H}{\eta}} \right]$$

$$\text{and } T_2 = \frac{A}{a} \sqrt{\frac{2}{g}} \left[\sqrt{\frac{H}{\eta}} - \sqrt{0} \right]$$

According to problem $T_1 = T_2$

$$\therefore \sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0 \Rightarrow \sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$$

2. (a) Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \frac{\text{N}}{\text{m}^2}$

Pressure due to liquid column

$$P_l = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$$

and velocity of water $v = \sqrt{2gh}$

$$\therefore v = \sqrt{\frac{2P_1}{\rho}} = \sqrt{\frac{2 \times 2 \times 10^5}{10^3}} = \sqrt{400} \text{ m/s}$$

3. (a) The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second

$$= Av = A\sqrt{2gh} \quad (i)$$

Volume of water flowing in per second

$$= 70 \text{ cm}^3/\text{s} \quad (ii)$$

From (i) and (ii), we get

$$A\sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2gh} = 70$$

$$\Rightarrow 1 \times \sqrt{2 \times 980 \times h} = 70$$

$$\therefore h = \frac{4900}{1960} = 2.5 \text{ cm}$$

4. (b) Horizontal range will be maximum when $h = \frac{H}{2} = \frac{90}{2} = 45 \text{ cm}$, i.e., hole 3.

5. (d) Let the density of water be ρ , then the force by escaping liquid on container = $\rho S(\sqrt{2gh})^2$

Acceleration of container

$$a = \frac{2\rho Sgh - \mu\rho Vg}{\rho V} = \left(\frac{2Sh}{V} - \mu\right)g$$

$$\text{Now } \mu = \frac{Sh}{V} \therefore a = \frac{Sh}{V}g$$

6. (d) The velocity of fluid at the hole

$$\text{is: } V_2 = \sqrt{\frac{2gh}{1 + (a^2/A^2)}}$$

Using continuity equation at the two cross-sections (1) and (2):

$$V_1 A = V_2 a \Rightarrow V_1 = \frac{a}{A} V_2$$

$$\Rightarrow \text{Acceleration of top surface} = -V_1 \frac{dV_1}{dh}$$

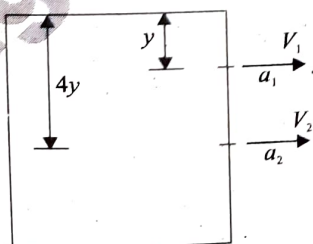
$$= -\frac{a}{A} V_2 \frac{d}{dh} \left(\frac{a}{A} V_2 \right)$$

$$a_1 = -\frac{a^2}{A^2} V_2 \frac{dV_2}{dh} = -\frac{a^2}{A^2} \sqrt{2gh} \sqrt{2g} \cdot \frac{1}{2\sqrt{h}}$$

$$\Rightarrow a_1 = \frac{-ga^2}{A^2}$$

7. (a) Velocity of efflux at a depth h is given by

$$v = \sqrt{2gh}$$



Volume of water flowing out per second from both the holes are equal.

$$\therefore a_1 v_1 = a_2 v_2$$

$$\text{or } L^2 \sqrt{2gy} = \pi R^2 \sqrt{2g(4y)}$$

$$\text{or } R = \frac{L}{\sqrt{2\pi}}$$

8. (b) From Bernoulli's theorem,

$$P_A + \frac{1}{2} dv_A^2 + dgh_A = P_B + \frac{1}{2} dv_B^2 + dgh_B$$

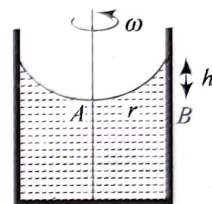
Here, $h_A = h_B$

$$\therefore P_A + \frac{1}{2} dv_A^2 = P_B + \frac{1}{2} dv_B^2$$

$$\Rightarrow P_A - P_B = \frac{1}{2} d[v_B^2 - v_A^2]$$

Now, $v_A = 0$, $v_B = r\omega$ and $P_A - P_B = hdg$

$$\therefore hdg = \frac{1}{2} dr^2 \omega^2 \text{ or } h = \frac{r^2 \omega^2}{2g}$$



Multiple Correct Answers Type

9. (b, c, d)

$$v = \sqrt{2gh}$$

So v is independent of area of orifice, area of cross-section of vessel and density of liquid. Hence options (b), (c) and (d) are correct.

10. (b, d)

$$V_A = \left(2g \frac{H}{3} \right)^{1/2}$$

$$V_B = \left(\frac{4gH}{3} \right)^{1/2} = \sqrt{2} \left(\frac{2gH}{3} \right)^{1/2} = \sqrt{2} V_A$$

$$Q_A = V_A(2a)$$

$$\text{and } Q_B = V_B(a) = V_A \sqrt{2}(a)$$

$$\frac{Q_A}{Q_B} = \sqrt{2}$$

Comprehension Type

11. (b) Using equation of continuity $A_1 v_1 = A_2 v_2$
 $(12 \text{ cm}^2) v_a = (6 \text{ cm}^2)(8.0 \text{ m/s})$

$$v_a = 4.0 \text{ m/s}$$

12. (d) Applying Bernoulli's principle between point A and C that are at the same horizontal level

$$\frac{1}{2} \rho \cdot V_A^2 + p_A = \frac{1}{2} \rho \cdot V_C^2 + p_{\text{atm}}$$

$$\Rightarrow p_A = (1.01 \times 10^5 \text{ N/m}^2) + \frac{1}{2} \times 13,600(8^2 - 4^2)$$

$$= 4.27 \times 10^5 \text{ N/m}^2$$

13. (c) By applying Bernoulli's equation between the point B and C and using equation of continuity
 $v_B = 8.57 \text{ m/s}$

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and $p_B = 3.70 \times 10^4$ Pascal

$$h = \frac{3.70 \times 10^4}{10 \times 13,600} = 272 \text{ mm}$$

- 14. (a)** Distance covered by plunger in time t is ut , the work performed by force F is $W = Fut$. The mass of liquid flowing out during time t is ρAut . If outflow velocity is v , from equation of continuity we have $Au = av$. The change in kinetic energy of the liquid during the time t is

$$\Delta KE = \rho Aut \left(\frac{v^2}{2} - \frac{u^2}{2} \right)$$

From work-energy theorem, we have

$$Fut = \rho Aut \left(\frac{v^2}{2} - \frac{u^2}{2} \right)$$

On eliminating u , we get $v^2 = \frac{2F}{\rho A} \times \frac{1}{1 - a^2/A^2}$

If $a \ll A$, then $v = \sqrt{\frac{2F}{\rho A}}$

Alternatively we may apply Bernoulli's theorem,

$$(P_0 + P) + \frac{1}{2} \rho u^2 + \rho gh = P_0 + \frac{1}{2} \rho v^2 + \rho gh$$

Where P_0 is the atmospheric pressure and P is the pressure due to external force F . As $a \ll A$, velocity inside cylinder is nearly

zero, we have $P = \frac{F}{A} = \frac{1}{2} \rho v^2$

$$\text{or } v = \sqrt{\frac{2F}{\rho A}}$$

- 15. (b)** Rate at which liquid is discharged,

$$-\frac{dV}{dt} = a \frac{dx}{dt} = av = a \sqrt{\frac{2F}{\rho A}}$$

$$\text{or } -\int_v^0 dV = a \sqrt{\frac{2F}{\rho A}} \int_0^t dt$$

$$\text{or } V = a \sqrt{\frac{2F}{\rho A}} t$$

$$\text{or } F = \frac{\rho A V^2}{2a^2 t^2}$$

Work done by force $W = F \frac{V}{A}$

$$= \frac{\rho A V^2}{2a^2 t^2} \times \frac{V}{A} = \frac{\rho V^3}{2a^2 t^2}$$

CHAPTER 5

DPP 5.1

Single Correct Answer Type

1. (d) $\text{Stress} = \frac{\text{Force}}{\text{area}}$

In the present case, force applied and area of cross-section of wires are the same, therefore stress has to be the same.

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

Since the Young's modulus of steel wire is greater than the copper wire, therefore strain in the case of steel wire is less than that in the case of copper wire.

2. (d) $l = \frac{FL}{AY} \therefore l \propto \frac{1}{r^2}$ (F, L and Y are constant)

$$\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 = (2)^2 = 4$$

3. (c) $l = \frac{FL}{\pi r^2 Y} \Rightarrow r^2 \propto \frac{1}{Y}$ (F, L and l are constant)

$$\frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7 \times 10^{10}}{12 \times 10^{10}}\right)^{1/2}$$

$$\Rightarrow r_2 = 1.5 \times \left(\frac{7}{12}\right)^{1/2} = 1.145 \text{ mm} \therefore \text{diameter} = 2.29 \text{ mm}$$

4. (d) $F = Y \times A \times \frac{l}{L} \Rightarrow F \propto \frac{r^2}{L}$ (Y and l are constants)

$$\frac{F_A}{F_B} = \left(\frac{r_A}{r_B}\right)^2 \times \left(\frac{L_B}{L_A}\right) = \left(\frac{2}{1}\right)^2 \times \left(\frac{2}{1}\right) = \frac{8}{1}$$

5. (d) As stress is shown on x -axis and strain on y -axis

$$\text{So we can say that } Y = \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\text{slope}}$$

So elasticity of wire P is minimum and of wire R is maximum.

6. (b) In ductile materials, yield point exists while in brittle material, failure would occur without yielding.

7. (c) $Y = \tan \theta$. According to the figure $\theta_A > \theta_B > \theta_C$

$$\text{i.e., } \tan \theta_A > \tan \theta_B > \tan \theta_C$$

$$\text{or } Y_A > Y_B > Y_C$$

$\therefore A, B$, and C graphs are for steel, brass and rubber respectively.

8. (b) Initial length (circumference) of the ring = $2\pi r$

Final length (circumference) of the ring = $2\pi R$

Change in length = $2\pi R - 2\pi r$

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

$$\text{Now Young's modulus } E = \frac{F/A}{1/L} = \frac{F/A}{(R-r)/r}$$

$$\therefore F = AE \left(\frac{R-r}{r} \right)$$

9. (d) If side of the cube is L , then $V = L^3 \Rightarrow \frac{dV}{V} = 3 \frac{dL}{L}$

$$\therefore \% \text{ change in volume} = 3 \times (\% \text{ change in length})$$

$$= 3 \times 1\% = 3\% \therefore \text{Bulk strain } \frac{\Delta V}{V} = 0.03$$

10. (a) If coefficient of volume expansion is α and rise in the temperature is $\Delta\theta$, then $\Delta V = V\alpha\Delta\theta \Rightarrow \frac{\Delta V}{V} = \alpha\Delta\theta$

$$\text{Volume elasticity } \beta = \frac{P}{\Delta V/V} = \frac{P}{\alpha\Delta\theta} \Rightarrow \Delta\theta = \frac{P}{\alpha\beta}$$

11. (c) For twisting, angle of shear $\phi \propto \frac{1}{L}$

i.e., if L is more, then ϕ will be small.

Multiple Correct Answers Type

12. (a, b, c, d)

According to Hooke's Law,
stress $\propto$ strain

$$\text{or } \frac{Mg}{A} \propto \frac{\Delta L}{L}$$

$$\text{or } Mg \propto \Delta L \quad (\text{as } A \text{ is constant})$$

$$\Delta L \propto Mg$$

Hence all the options (a), (b), (c) and (d) are correct.

13. (a, d)

$$Y = \frac{\text{long stress}}{\text{long strain}}; B = \frac{\text{normal stress}}{\text{volume strain}}$$

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}}$$

As strain is dimensionless, hence Y, B and η possess the units of pressure.

Y depends on the material of the body only.

Y is related with length elasticity of material.

So options (a) and (d) are correct.

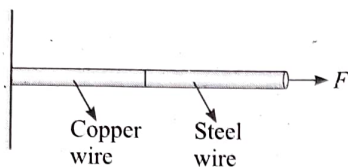
14. (c, d)

It is clear from the two graphs, the ultimate tensile strength for material (ii) is greater, hence material (ii) is elastic over larger region as compared to material (i). For material (ii) fracture point is nearer, hence it is more brittle.

15. (a, d)

Consider the diagram where a deforming force F is applied to the combination.

$$\text{For steel wire, } Y_{\text{steel}} = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\text{Strain}}$$



where F is tension in each wire and A is cross-section area of each wires.

As F and A are the same for both the wire, hence stress will be the same for both the wire.

$$(\text{Strain})_{\text{steel}} = \frac{\text{Stress}}{Y_{\text{steel}}}, (\text{Strain})_{\text{copper}} = \frac{\text{Stress}}{Y_{\text{copper}}}$$

As, $Y_{\text{steel}} \neq Y_{\text{copper}}$

Hence, the two wires will have different strain.

DPP 5.2

Single Correct Answer Type

1. (d) $U = \frac{1}{2} \left(\frac{YA}{L} \right) l^2 \therefore U \propto l^2$

$$\frac{U_2}{U_1} = \left(\frac{l_2}{l_1} \right)^2 = \left(\frac{10}{2} \right)^2 = 25 \Rightarrow U_2 = 25U_1$$

i.e., potential energy of the spring will be 25V

2. (a) $W = \frac{1}{2} Fl \therefore W \propto l$ (F is constant)

$$\therefore \frac{W_1}{W_2} = \frac{l_1}{l_2} = \frac{l}{2l} = \frac{1}{2}$$

3. (b) $W = \frac{1}{2} \times F \times l = \frac{1}{2} mgl$
 $= \frac{1}{2} \times 10 \times 10 \times 1 \times 10^{-2} = 0.05 \text{ J}$

4. (b) $U = \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume} = \frac{1}{2} \times \frac{F^2 \times A \times L}{A^2 \times Y}$
 $= \frac{1}{2} \times \frac{F^2 L}{AY} = \frac{1}{2} \times \frac{(50)^2 \times 0.2}{1 \times 10^{-4} \times 1 \times 10^{11}} = 2.5 \times 10^{-5} \text{ J}$

5. (c) Work done in stretching a wire

$$W = \frac{1}{2} Fl = \frac{1}{2} \times 10 \times 0.5 \times 10^{-3} = 2.5 \times 10^{-3} \text{ J}$$

Work done to displace it through 1.5 mm

$$W = F \times l = 5 \times 10^{-3} \text{ J}$$

The ratio of above two work = 1 : 2

6. (d) At extension l_1 , the stored energy = $\frac{1}{2} Kl_1^2$

At extension l_2 , the stored energy = $\frac{1}{2} Kl_2^2$

Work done in increasing its extension from l_1 to l_2

$$= \frac{1}{2} K(l_2^2 - l_1^2)$$

7. (b) $K = \frac{F}{x} = \frac{40}{2 \times 10^{-2}} = 0.2 \text{ N/m}$

$$\text{Work done} = \frac{1}{2} Kx^2 = \frac{1}{2} \times (0.2) \times (0.05)^2 = 2.5 \text{ J}$$

8. (b) $U = \frac{1}{2} Fl = \frac{F^2 L}{2AY} \cdot U \propto \frac{L}{r^2}$ (F and Y are constant)

$$\therefore \frac{U_A}{U_B} = \left(\frac{L_A}{L_B} \right) \times \left(\frac{r_A}{r_B} \right)^2 = (3) \times \left(\frac{1}{2} \right)^2 = \frac{3}{4}$$

9. (b) $F = - \left(\frac{dU}{dx} \right)$

In the region BC slope of the graph is positive

$\therefore F = \text{negative}$, i.e., force is attractive in nature

In the region AB slope of the graph is negative

$\therefore F = \text{positive}$, i.e. force is repulsive in nature

10. (d) $\tau = C \cdot \theta = \frac{\pi \eta r^4 \theta}{2L} = \text{Constant}$

$$\Rightarrow \frac{\pi \eta r^4 (\theta - \theta_0)}{2l} = \frac{\pi \eta (r/2)^4 (\theta_0 - \theta')}{2(l/2)}$$

$$\Rightarrow \frac{(\theta - \theta_0)}{2} = \frac{\theta_0}{16} \Rightarrow \theta_0 = \frac{8}{9} \theta$$

11. (d) $\eta = \frac{F/A}{x/L} \Rightarrow x = \frac{L}{\eta} \times \frac{F}{A}$

If η and F are constant, then $x \propto \frac{L}{A}$

For maximum displacement area at which force applied should be minimum and vertical side should be maximum, this is given in the R position of rectangular block.

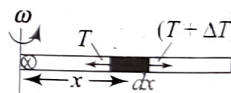
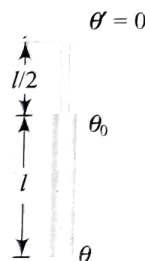
12. (a) $-\int_T^0 \Delta T = \int_0^l \frac{m}{l} dx \omega^2 x \quad T = \frac{m}{l} \omega^2 \frac{x^2}{2}$

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$$

$$\Delta l = \frac{l}{AY} \int_0^l \frac{m \omega^2 x^2}{2} dx$$

$$\Delta l = \frac{m \omega^2 l^3}{l 6AY} \Rightarrow \Delta l = \frac{\rho \omega^2 l^3}{6Y}$$

$$\Delta l \propto \omega^2 \Rightarrow \omega^2 = 2\omega_1$$



Multiple Correct Answers Type

13. (b, c, d)

As the length of wire is doubled on stretching, hence $\Delta L = L$

$$\text{So strain} = \frac{\Delta L}{L} = 1$$

As $Y = \frac{\text{stress}}{\text{strain}}$ and strain = 1, so $Y = \text{stress}$

Elastic energy per unit volume,

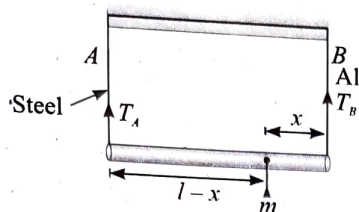
$$E = \frac{1}{2} Y(\text{strain})^2 = \frac{1}{2} Y$$

$$\therefore Y = 2E$$

14. (b, d)

Let the mass is placed at x from the end B .

Let T_A and T_B be the tensions in wire A and wire B respectively.



For the rotational equilibrium of the system,

$$\Sigma \tau = 0 \text{ (Total torque = 0)}$$

$$\Rightarrow T_B x - T_A (l - x) = 0$$

$$\Rightarrow \frac{T_B}{T_A} = \frac{l - x}{x}$$

$$\text{Stress in wire A} = S_A = \frac{T_A}{a_A}$$

$$\text{Stress in wire B} = S_B = \frac{T_B}{a_B}$$

where a_A and a_B are cross-sectional areas of wires A and B respectively.

By question $a_B = 2a_A$

Now, for equal stress $S_A = S_B$

$$\Rightarrow \frac{T_A}{a_A} = \frac{T_B}{a_B} \Rightarrow \frac{T_B}{T_A} = \frac{a_B}{a_A} = 2$$

$$\Rightarrow \frac{l - x}{x} = 2 \Rightarrow \frac{l}{x} - 1 = 2$$

$$\Rightarrow x = \frac{l}{3} \Rightarrow l - x = l - l/3 = \frac{2l}{3}$$

Hence, mass m should be placed closer to B.

For equal strain, $(\text{strain})_A = (\text{strain})_B$

$$\Rightarrow \frac{Y_A}{S_A} = \frac{Y_B}{S_B} \text{ (where } Y_A \text{ and } Y_B \text{ are Young's moduli)}$$

$$\Rightarrow \frac{Y_{\text{steel}}}{T_A/a_A} = \frac{Y_{\text{Al}}}{T_B/a_B}$$

$$\Rightarrow \frac{Y_{\text{steel}}}{Y_{\text{Al}}} = \frac{T_A}{T_B} \times \frac{a_B}{a_A} = \left(\frac{x}{l-x} \right) \left(\frac{2a_A}{a_A} \right)$$

$$\Rightarrow \frac{200 \times 10^9}{70 \times 10^9} = \frac{2x}{l-x} \Rightarrow \frac{20}{7} = \frac{2x}{l-x}$$

$$\Rightarrow \frac{10}{7} = \frac{x}{l-x} \Rightarrow 10l - 10x = 7x$$

$$\Rightarrow 17x = 10l \Rightarrow x = \frac{10l}{17}$$

$$\Rightarrow l - x = l - \frac{10l}{17} = \frac{7l}{17}$$

Hence, mass m should be placed closer to wire A.

15. (b, c)

$$(\text{stress})_S = \frac{F}{2A}, (\text{stress})_{\text{Cu}} = \frac{F}{A}$$

$$\text{Given that: } \frac{Y_S}{Y_{\text{Cu}}} = \frac{2}{1}$$

$$\frac{(\text{strain})_S}{(\text{strain})_{\text{Cu}}} = \frac{(\text{stress})_S/Y_S}{(\text{stress})_{\text{Cu}}/Y_{\text{Cu}}}$$

$$= \frac{(\text{stress})_S}{(\text{stress})_{\text{Cu}}} \times \frac{Y_{\text{Cu}}}{Y_S} = \left(\frac{F/2A}{F/A} \right) \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{L_S/\Delta L_S}{L_{\text{Cu}}/\Delta L_{\text{Cu}}} = \frac{1}{4} \text{ or } \frac{L_S}{L_{\text{Cu}}} \times \frac{\Delta L_{\text{Cu}}}{\Delta L_S} = \frac{1}{4}$$

$$\frac{25}{50} \times \frac{\Delta L_{\text{Cu}}}{\Delta L_S} = \frac{1}{4} \text{ or } \frac{\Delta L_{\text{Cu}}}{\Delta L_S} = \frac{1}{2}$$

So options (b) and (c) are correct.

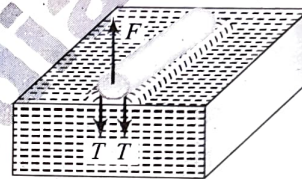
DPP 5.3

Single Correct Answer Type

1. (a) Because film tries to cover minimum surface area.

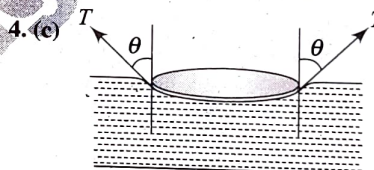
$$2. (c) T = \frac{F}{2l} = \frac{728}{2 \times 5}$$

$$\therefore T = 72.8 \text{ dyne/cm}$$



3. (d) Cohesive force > Adhesive force, so shape of liquid surface near the solid would be convex.

For example mercury surface in glass capillary is convex.



Weight of metal disc = total upward force
= upthrust force + force due to surface tension
= weight of displaced water + $T \cos \theta (2\pi r)$
= $W + 2\pi r T \cos \theta$

5. (a) Energy needed = Increment in surface energy
= (surface energy of n small drops) - (surface energy of one big drop)
= $n4\pi r^2 T - 4\pi R^2 T = 4\pi T(nr^2 - R^2)$

$$6. (c) W = 8\pi T(r_2^2 - r_1^2) = 8\pi T \left[\left(\frac{2}{\sqrt{\pi}} \right)^2 - \left(\frac{1}{\sqrt{\pi}} \right)^2 \right]$$

$$\therefore W = 8 \times \pi \times 30 \times \frac{3}{\pi} = 720 \text{ erg}$$

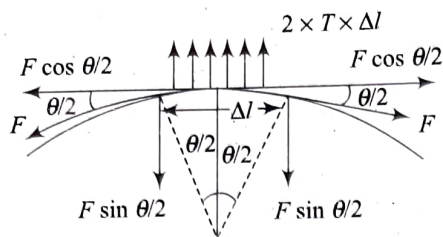
7. (d) As volume remains constant, therefore $R = n^{1/3} r$

$$\frac{\text{Surface energy of one big drop}}{\text{Surface energy of } n \text{ drop}} = \frac{4\pi R^2 T}{n \times 4\pi r^2 T}$$

$$\frac{R^2}{nr^2} = \frac{n^{2/3} r^2}{nr^2} = \frac{1}{n^{1/3}} = \frac{1}{(1000)^{1/3}} = \frac{1}{10}$$

8. (b) Increment in area of soap film = $A_2 - A_1$
= $2 \times [(10 \times 0.6) - (10 \times 0.5)] \times 10^{-4} = 2 \times 10^{-4} \text{ m}^2$
Work done = $T \times \Delta A$
= $7.2 \times 10^{-2} \times 2 \times 10^{-4} = 1.44 \times 10^{-5} \text{ J}$

9. (d) Suppose tension in thread is F , then for small part Δl of thread



$$\Delta l = r\theta \text{ and } 2F \sin \theta/2 = 2T\Delta l = 2TR\theta$$

$$\Rightarrow F = \frac{TR\theta}{\sin \theta/2} = \frac{TR\theta}{\theta/2} = 2TR(\sin \theta/2 \approx \theta/2)$$

10. (c) Rise in temperature, $\Delta\theta = \frac{3T}{JSd} \left(\frac{1}{r} - \frac{1}{R} \right)$

$$\therefore \Delta\theta = \frac{3T}{J} \left(\frac{1}{r} - \frac{1}{R} \right) \quad (\text{For water } S = 1 \text{ and } d = 1)$$

$$\Delta P \propto \frac{1}{t}$$

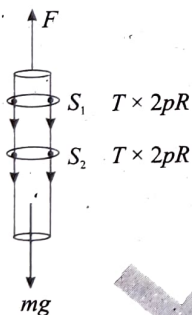
11. (c) The force exerted by a film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

12. (b) Figure shows one of the legs of the mosquito landing upon the water surface. Therefore, $T2\pi a \times 8 = W = \text{weight of the mosquito}$.



13. (b) The free body diagram of the capillary tube is as shown in the figure. Net force F required to hold tube is

$F = \text{force due to surface tension at cross-section}$



Free body diagram of capillary tube

$$(S_1 + S_2) + \text{weight of tube} = (2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$$

Here $R = 2\text{mm} = 2 \times 10^{-3}\text{m}$, $T = 0.1\text{ N/m}$ and $m = \pi \times 10^{-3}\text{ kg}$
 $f = 4\pi (2 \times 10^{-3}) (0.1) + (\pi \times 10^{-3}) \times 10 = 10.8\pi \times 10^{-3}\text{ N}$
 $= 10.8\pi\text{ mN}$

Fill in the Blanks Type

14. $T = \frac{F}{l} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$

15. Force on each side = $2TL$ (due to two surfaces)

$$\therefore \text{Force on the frame} = 4(2TL) = 8TL$$

16. The total length of the circular plate on which the force will act = $2\pi R$

$$\text{Force to pull} = 2\pi RT = 2 \times \pi \times 750\pi \text{ dynes}$$

17. $T = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{-2}} = 0.1\text{ N/m}$

18. Force required to separate the plates

$$F = \frac{2TA}{t} = \frac{2 \times 70 \times 10^{-3} \times 10^{-2}}{0.05 \times 10^{-3}} = 28\text{ N}$$

19. Net force on stick = $F_1 - F_2 = (T_1 - T_2)l$
 $= (0.07 - 0.06)l = 0.01 \times 2 = 0.02\text{ N}$

20. Force required, $F = 2\pi rT = 2\pi \times 2 \times 70 = 280\pi \text{ dyne}$

21. When two droplets merge with each other, their surface energy decreases.

$W = T(\Delta A) = (\text{negative})$, i.e. energy is released.

22. $W = T \times \Delta A = 5 \times 2 \times (0.02)$ (Film has two free surfaces)
 $= 2 \times 10^{-1}\text{ J}$

23. $W = 8\pi R^2T \therefore W \propto R^2$ (T is constant)

If radius becomes double, then work done will become four times.

24. Surface energy of combined drop will be lowered, so excess surface energy will raise the temperature of the drop.

25. Surface energy = surface tension $\times$ increment in area
 $= T \times A$

DPP 5.4

Single Correct Answer Type

1. (a) $\Delta P = \frac{4T}{r} \therefore \Delta P \propto \frac{1}{r}$

As radius of soap bubble increases with time $\therefore \Delta P \propto \frac{1}{r}$

2. (c) Excess pressure inside soap bubble is inversely proportional to the radius of bubble, i.e., $\Delta P \propto \frac{1}{r}$

This means that bubbles A and C possess greater pressure inside it than B. So the air will move from A and C towards B.

3. (c) Outside pressure = 1 atm

Pressure inside first bubble = 1.01 atm

Pressure inside second bubble = 1.02 atm

Excess pressure $\Delta P_1 = 1.01 - 1 = 0.01\text{ atm}$

Excess pressure $\Delta P_2 = 1.02 - 1 = 0.02\text{ atm}$

$$\Delta P \propto \frac{1}{r} \Rightarrow r \propto \frac{1}{\Delta P} \Rightarrow \frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{0.02}{0.01} = \frac{2}{1}$$

Since $V = \frac{4}{3}\pi r^3 \therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2} \right)^3 = \left(\frac{2}{1} \right)^3 = \frac{8}{1}$

4. (b) Pressure at half the depth = $P_0 + \frac{h}{2}dg$

Pressure at the bottom = $P_0 + hdg$

According to given condition,

$$P_0 + \frac{h}{2}dg = \frac{2}{3}(P_0 + hdg)$$

$$\Rightarrow 3P_0 + \frac{3h}{2}dg = 2P_0 + 2hdg$$

$$\Rightarrow h = \frac{2P_0}{dg} = \frac{2 \times 10^5}{10^3 \times 10} = 20\text{ m}$$

5. (c) Excess pressure inside the air bubble $= \frac{2T}{r}$

$$\Rightarrow P_{\text{in}} - P_{\text{out}} = \frac{2T}{r} = \frac{2 \times 70 \times 10^{-3}}{0.1 \times 10^{-3}} = 1400 \text{ Pa}$$

$$\Rightarrow P_{\text{in}} = 1400 + 1.013 \times 10^5 = 1.027 \times 10^5 \text{ Pa}$$

6. (c) Angle of contact is acute.

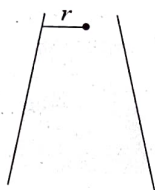
7. (b) $h = \frac{2T \cos \theta}{rdg} \therefore h \propto \frac{1}{r}$. So the graph between h and r will be rectangular hyperbola.

8. (b) Because if the length available is less than required, then water will rise up to available height and adjust its radius of curvature.

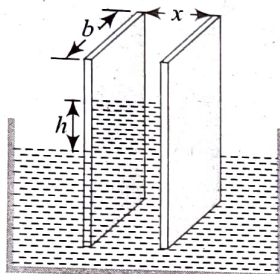
9. (b) $2\pi r T = \pi r^2 \rho g h \Rightarrow h = \frac{2T}{r \rho g}$

But weight of liquid in tapered tube is more than uniform tube of radius r , then in order to balance $h < h_1$

$$h < \frac{2T}{r \rho g}$$



10. (b)



Let the width of each plate is b and due to surface tension liquid will rise up to height h , then upward force due to surface tension $= 2Tb \cos \theta$ (i)

Weight of the liquid rises in between the plates $= Vdg = (bxh)dg$ (ii)

Equating (i) and (ii) we get, $2T \cos \theta = bxh dg$

$$\therefore h = \frac{2T \cos \theta}{xdg}$$

11. (d) $\rho g h \pi r^2 = 2\pi r S \cos \theta$

$$\Rightarrow r = \frac{2S \cos \theta}{\rho g h} = \frac{2 \times 1 \times 0.5}{10^3 \times 10 \times 10} = 10^{-6} \text{ m}$$

12. (a) Let the container is dipped to depth h , so the contact angle becomes θ

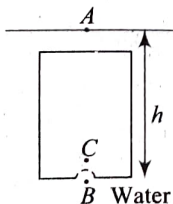
$$P_A = P_0$$

$$P_B = P_0 + \rho g h$$

$$P_C = P_0 + \rho g h - \frac{2T}{r} \cos \theta = P_0$$

$$\cos \theta = \frac{\rho g h r}{2T} \leq 1$$

$$h \leq \frac{2T}{\rho g r} \Rightarrow h_{\text{max}} = \frac{2T}{\rho g r}$$



Multiple Correct Answers Type

13. (a, d)

Work done in forming a soap bubble of radius R ,

$$W = T \Delta A = T \times 2 \times 4\pi R^2 = 8\pi R^2 T$$

Work done in doubling the radius of a soap bubble of radius R $= T \times 2 \times 4\pi (2R)^2 - 8\pi R^2 T = 24\pi R^2 T$

Excess pressure inside a soap bubble of radius $R = \frac{4T}{R}$. So (a) and (d) are the required options.

14. (a, c, d)

If cohesive force $>$ adhesive force, then resultant force on a liquid molecule in contact with the solid surface will be directed into the liquid at some angle w.r.t. solid surface. This implies that liquid surface will be convex and angle of contact will be greater than 90° , i.e., obtuse.

When $\theta > 90^\circ$, $\cos \theta$ is -ve and so h will be -ve ($\because h \propto \cos \theta$), i.e., there will be capillary descent.

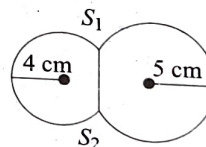
Fill in the Blanks Type

15. Tangent drawn at point of contact makes 0° with wall of container.

16. $h \propto \frac{1}{r} \therefore r_1 h_1 = r_2 h_2 \Rightarrow h_2 = \frac{r_1 h_1}{r_2} = 2.4 \text{ mm}$

17. $S = \frac{rhdg}{2 \cos \theta} \Rightarrow \text{Pressure difference} = hdg = \frac{2S}{r} \cos \theta$

18. $r = \frac{r_1 r_2}{r_1 - r_2} = \frac{5 \times 4}{5 - 4} = 20 \text{ cm}$



19. Mass of liquid in capillary tube

$$M = \pi R^2 H \times \rho \therefore M \propto R^2 \times \left(\frac{1}{R}\right) \text{ (As } H \propto 1/R)$$

$\therefore M \propto R$. If radius becomes double, then mass will become twice.

20. Ascent formula $h = \frac{2T \cos \theta}{rdg}$

$$\Rightarrow \frac{h_1}{h_2} = \frac{T_1}{T_2} \times \frac{d_2}{d_1}$$

(r , θ and g are constants)

$$\Rightarrow \frac{h_1}{h_2} = \frac{60}{50} \times \frac{0.6}{0.8} = \frac{9}{10}$$

21. $\frac{4T}{R} = hdg \therefore T = \frac{Rhdg}{4}$

$$T = \frac{10^{-2} \times 2 \times 10^{-3} \times 0.8 \times 10^3 \times 9.8}{4} = 3.9 \times 10^{-2} \text{ N/m}$$

DPP 5.5

Single Correct Answer Type

- (b) (i) and (iii) are true but (ii) is false as we know that viscosity in gases is about 100 times less than viscosity in liquids.
- (d) When we move from the centre to circumference, the velocity of liquid goes on decreasing and finally becomes zero.
- (c) When the pebble is falling through the viscous oil, the viscous force is $F = 6\pi\eta rv$ where r is the radius of the pebble, v is instantaneous speed, and η is the coefficient of viscosity. As the force is variable, hence acceleration is also variable so $v-t$ graph will not be straight line. First velocity increases and then becomes constant known as terminal velocity.
- (c) As the ball ascends upwards in addition to weight, air friction also acts downwards. Hence the initial magnitude of acceleration will be greater than magnitude of acceleration due to gravity. So the only possible option is (c).
- (c) The coefficient of viscosity is the ratio of tangential stress on top surface of film (exerted by block) to that of velocity gradient (vertically downwards) of film. Since mass m moves with constant velocity, the string exerts a force equal to mg on plate towards right. Hence oil shall exert tangential force mg on plate towards left.

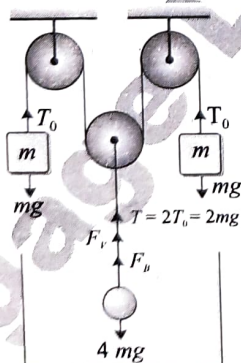
$$\eta = \frac{F/A}{(v-0)/\Delta x} = \frac{125 \times 1000/10 \times 20}{(5-0)/.02} = 2.5 \text{ dyne-s/cm}^2$$

6. (a) Viscous force = $mg \sin \theta$

$$\therefore \eta A \frac{v}{t} = mg \sin \theta \text{ or } \eta a^2 \frac{v}{t} = a^2 \rho g \sin \theta$$

$$\eta = \frac{t \rho g \sin \theta a}{v}$$

7. (b) From the free body diagram of the sphere:



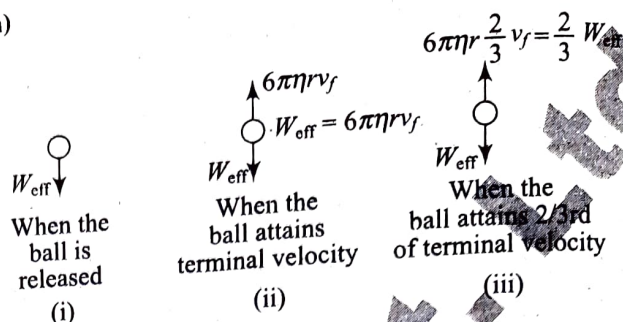
$$F_v = 4mg - 2mg = F_B$$

$$\Rightarrow F_v = 2mg - F_B$$

$$\Rightarrow 6\pi\eta rV = \frac{4}{3}\pi r^3 \left(\frac{\sigma}{2} - \rho \right) g \quad \left(\text{since } 4m = \frac{4}{3}\pi r^3 \times \sigma \right)$$

$$\Rightarrow V = \frac{2}{9} r^2 \frac{(\sigma - 2\rho)g}{\eta}$$

8. (a)



When the ball is just released, the net force on ball is W_{eff} ($= mg - \text{buoyant force}$)
The terminal velocity ' v_f ' of the ball is attained when net force on the ball is zero.
 $\therefore$ Viscous force $6\pi\eta r = W_{\text{eff}}$

When the ball acquires $\frac{2}{3}$ rd of its maximum velocity v_f

The viscous force is $= \frac{2}{3} W_{\text{eff}}$

Hence net force is $W_{\text{eff}} - \frac{2}{3} W_{\text{eff}} = \frac{1}{3} W_{\text{eff}}$

$\therefore$ Required acceleration is $= \frac{a}{3}$

9. (c) The only force acting on the body is the viscous force

$$\text{Here } m \frac{dv}{dx} = -6\pi\eta rv = -rv$$

$$\Rightarrow \int_0^x m dv = \int_0^x -r dx \Rightarrow x = \frac{mv}{r}$$

10. (b) Initially the terminal velocity V of sphere of radius a is

$$W_{\text{eff}} = 6\pi\eta aV \quad (1)$$

(W_{eff} = weight - buoyant force)

As the radius is doubled, mass is increased to 8 times and new terminal velocity will be

$$8W_{\text{eff}} = 6\pi\eta 2aV' \quad (2)$$

From 1 and 2 $V' = 4V$

11. (b) Relative to liquid, the velocity of sphere is $2v_0$ upwards.

$\therefore$ viscous force on sphere = $6\pi\eta r 2v_0$ downward
 $= 12\pi\eta r v_0$ downward

12. (d) Let v be the velocity of the movable plate and F is equal to viscous force

$$F = \left[\eta_1 \frac{v}{h_1} + \eta_2 \frac{v}{h - h_1} \right] A$$

$$\Rightarrow \frac{dF}{dh_1} = 0 \quad \therefore h_1 = \frac{h}{3}$$

13. (b) At equilibrium

$$mg = 6\pi\eta rv \text{ or } \rho \frac{4\pi}{3} r^3 g = 6\pi \eta rv$$

$$\therefore \frac{v_r}{v_{2r}} = \frac{(r)^2}{(2r)^2} \text{ or } v_{2r} = (v_r) \times 4 = 4 \text{ cm/s}$$

$$14. (a) \quad 6\pi\eta Rv_1 = \frac{4}{3}\pi R^3(\rho_1 - \sigma)$$

$$6\pi\eta Rv_2 = \frac{4}{3}\pi R^3(\rho_2 - \sigma)$$

$$\therefore \frac{v_2}{v_1} = \left(\frac{\rho_2 - \sigma}{\rho_1 - \sigma} \right)$$

Multiple Correct Answers Type

15. (b, c)

Streamline flow is more likely for liquids having low density. We know that greater the coefficient of viscosity of a liquid, more will be velocity gradient hence each line of flow can be easily differentiated.

16. (b, c, d)

$$F_{\text{eff}} = 6\pi\eta r \times 20 \quad (1)$$

Where $6\pi\eta r \times 20$ = viscous force acting on the sphere in upward direction when it is moving downwards with a constant velocity of 20 m/s.

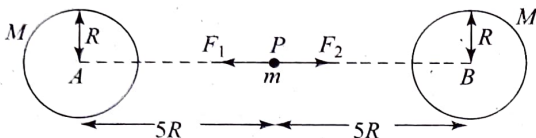
When the sphere is taken in a gravity-free hall having air at the same pressure and pushed down at a speed of 20 m/s, it will experience a force F_{eff} . In upward direction it will generate an initial acceleration of 9.8 m/s^2 in it in upward direction. Due to this force, its downward velocity will go on decreasing and therefore upward viscous force ($\propto$ velocity) also. After some time, force as well as velocity will become zero and the body will eventually stop. So options (b), (c) and (d) are correct.

CHAPTER 6

DPP 6.1

Subjective Type

1. Let the mass and radius of each identical heavy sphere be M and R respectively. An object of mass m be placed at the mid-point P of the line joining their centres.



Force acting on the object placed at the mid-point,

$$F_1 = F_2 = \frac{GMm}{(5R)^2}$$

The direction of forces are opposite, therefore net force acting on the object is zero.

To check the stability of the equilibrium, we displace the object through a small distance x towards sphere A.

Now, force acting towards sphere, A

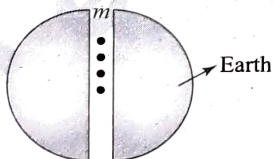
$$F_1' = \frac{GMm}{(5R - x)^2}$$

Force acting towards sphere B, $F_2' = \frac{GMm}{(5R + x)^2}$

As $F_1' > F_2'$, therefore a resultant force $(F_1' - F_2')$ acts on the object towards sphere A, therefore object start to move towards sphere A and hence equilibrium is unstable.

2. Yes, a body can have inertia (i.e., mass) but no weight. Everybody always have inertia (i.e., mass) but its weight (mg) can be zero, when it is taken at the centre of the earth or during free fall under gravity.

e.g., In the tunnel through the centre of the earth, the object moves only due to inertia at the centre while its weight becomes zero.



Single Correct Answer Type

3. (c) As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law.

Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth, will not be approximately circular, since the major gravitational force on mercury is due to the sun.

4. (b) $F = 0$ when $0 \leq r \leq R_1$

Because intensity is zero inside the cavity.

F increase when $R_1 \leq r \leq R_2$

$$F \propto \frac{1}{r^2} \text{ when } r > R_2$$

5. (c) Kepler's law $T^2 \propto R^3$
 6. (a) During path DAB planet is nearer to sun as compared with path BCD . So time taken in travelling DAB is less than that for BCD because velocity of planet will be more in region DAB .
 7. (c) Because distance of point C is maximum from the sun.
 8. (a) As the earth is revolving around the sun in a circular motion due to gravitational attraction. The force of attraction will be of radial nature, i.e., angle between position vector r and force F is zero. So, torque $= |\mathbf{r} \times \mathbf{F}| = r F \sin 0^\circ = 0$
 9. (d) Asteroids are also being acted upon by central gravitational forces, hence they are moving in circular orbits like planets and obey Kepler's laws.

10. (c) Force on B due to A $F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to

$$C = F_{BC} = \frac{GMm}{(BC)^2} \text{ towards BC}$$

As $(BC) = 2AB$

$$\Rightarrow F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$$

Hence, m will move towards BA (i.e., $2M$)

Multiple Correct Answers Type

11. (a, c)

If the law of gravitation becomes an inverse cube law, then we can write, for a planet of mass m revolving around the sun of mass M ,

$$F = \frac{GMm}{a^3} = \frac{mv^2}{a} \text{ (where } a \text{ is radius of orbiting planet)}$$

$$\Rightarrow v = \text{orbital speed} = \frac{\sqrt{GM}}{a} \Rightarrow v \propto \frac{1}{a}$$

Time period of revolution of a planet

$$T = \frac{2\pi a}{v} = \frac{2\pi a}{\frac{\sqrt{GM}}{a}} = \frac{2\pi a^2}{\sqrt{GM}}$$

$$\Rightarrow T^2 \propto a^4$$

Hence, orbit will not be elliptical.

[for elliptical orbit $T^2 \propto a^3$]

$$\text{As force } F = \left(\frac{GM}{a^3}\right)m = g'm$$

$$\text{where } g' = \frac{GM}{a^3}$$

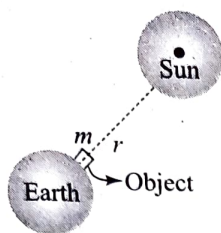
As g' , acceleration due to gravity, is constant, hence path followed by a projectile will be approximately parabolic (as $T \propto a^2$)

2. (a, c, d)

Given, $G' = 10G$

Consider the adjacent diagram.

Force on the object due to the earth



$$\begin{aligned}
 &= \frac{G'M_em}{R^2} = \frac{10GM_em}{R^2} \quad [\because G' = 10G \text{ given}] \\
 &= 10 \left(\frac{GM_em}{R^2} \right) \\
 &= (10g)m = 10 \text{ mg} \quad [\because g = \frac{GM_e}{R^2}] \quad (i)
 \end{aligned}$$

Force on the object due to the sun $F = \frac{GM'_sm}{r^2}$

$$= \frac{G(M_s)m}{10r^2} \quad [\because M'_s = \frac{M_s}{10} \text{ (given)}]$$

As $r \gg R$ (radius of the earth) $\Rightarrow F$ will be very small.

So, the effect of the sun will be neglected.

Now, as $g' = 10g$ [from Eq. (i)]Hence, weight of person $= mg' = 10mg$

i.e., gravity pull on the person will increase. Due to it, walking on ground would become more difficult.

Critical velocity, v_c is proportional to g , i.e.,

$$v_c \propto g$$

As, $g' > g \Rightarrow v'_c > v_c$

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the aeroplanes will have to travel much faster.

13. (c, d)

We know that gravitational force between the earth and the sun.

$$F_G = \frac{GMm}{r^2}$$

where M is the mass of the sun and m is the mass of the earth.

When G decreases with time, the gravitational force F_G will become weaker with time. As F_G is changing with time. Due to it, the earth will be going around the sun not strictly in closed orbit and radius also increases, since the attraction force is getting weaker.

Hence, after long time the earth will leave the solar system.

14. (a, c, d)

$$\text{Given, } F_1 = -F_2 = \frac{-r_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2} \right)^n$$

$$\Rightarrow r_{12} = r_1 - r_2$$

$$\text{Acceleration due to gravity, } g = \frac{|F|}{\text{mass}}$$

$$= \frac{GM_0^2 (m_1 m_2)^n}{r_{12}^2 (M_0)^{2n}} \times \frac{1}{(\text{mass})}$$

Since, g depends upon position vector, hence it will be different for different objects. As g is not constant, hence constant of proportionality will not be constant in Kepler's third law.

Hence, Kepler's third law will not be valid.

As the force is of central nature.

$$[\because \text{Force} \propto \frac{1}{r^2}]$$

Hence, first two Kepler's laws will be valid.

For negative n ,

$$\begin{aligned}
 g &= \frac{GM_0^2 (m_1 m_2)^{-n}}{r_{12}^2 (M_0)^{-2n}} \times \frac{1}{(\text{mass})} \\
 &= \frac{GM_0^{2(1+n)} (m_1 m_2)^{-n}}{r_{12}^2 (\text{mass})} \\
 g &= \frac{GM_0^2}{r_{12}^2} \left(\frac{M_0^2}{m_1 m_2} \right)^n \times \frac{1}{\text{mass}}
 \end{aligned}$$

As $M_0 > m_1$ or m_2
 $g > 0$, hence in this case situation will reverse, i.e., object lighter than water will sink in water.

Fill in the Blanks Type

$$15. \frac{T_{\text{mercury}}}{T_{\text{earth}}} = \left(\frac{r_{\text{mercury}}}{r_{\text{earth}}} \right)^{3/2} = \left(\frac{6 \times 10^{10}}{1.5 \times 10^{11}} \right)^{3/2} = \frac{1}{4} \text{ (approx.)}$$

$$\therefore T_{\text{mercury}} = \frac{1}{4} \text{ year}$$

16. For revolution of planet centripetal force is provided by gravitational force of attraction

$$m\omega^2 R \propto R^{-5/2} \Rightarrow \frac{1}{T^2} \propto R^{-7/2} \Rightarrow T^2 \propto R^{7/2}$$

17. Angular momentum remains constant

$$mv_1 d_1 = mv_2 d_2 \Rightarrow v_2 = \frac{v_1 d_1}{d_2}$$

18. According to Kepler's law $T^2 \propto R^3$ If N is the frequency, then $N^2 \propto (R)^{-3}$

$$\text{or } \frac{N_2}{N_1} = \left(\frac{R_2}{R_1} \right)^{-3/2} \Rightarrow \frac{R_1}{R_2} = \left(\frac{N_2}{N_1} \right)^{2/3}$$

DPP 6.2

Subjective Type

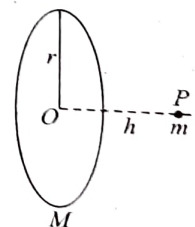
1. Consider the diagram, in which a system consisting of a ring and a point mass is shown.

Gravitational field intensity at point P at a distance h along the normal through the centre of a circular ring of mass M and

$$\text{radius } r \text{ is given by } E = \frac{GMh}{(r^2 + h^2)^{3/2}}$$

Gravitational force acting on an object of mass m

$$F = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

(along PO) (i)When mass is displaced up to distance $2h$, then

$$F' = \frac{GMm \times 2h}{[r^2 + (2h)^2]^{3/2}} \quad [\because h = 2r]$$

$$= \frac{2GMmh}{(r^2 + 4h^2)^{3/2}} \quad (ii)$$

When $h = r$, then from Eq. (i)

$$F = \frac{GMm \times r}{(r^2 + r^2)^{3/2}} \Rightarrow F = \frac{GMm}{2\sqrt{2}r^2}$$

and $F' = \frac{2GMmr}{(r^2 + 4r^2)^{3/2}} = \frac{2GMm}{5\sqrt{5}r^2}$

[From Eq. (ii) substituting $h = r$]

$$\therefore \frac{F'}{F} = \frac{4\sqrt{2}}{5\sqrt{5}} \Rightarrow F' = \frac{4\sqrt{2}}{5\sqrt{5}} F$$

Single Correct Answer Type

2. (a) Mass of the ball always remain constant. It does not depend upon the acceleration due to gravity
3. (d) If we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. In this case acceleration due to gravity $g = 0$, at the centre. It is not so, if the earth is considered as a sphere of nonuniform density, in that case value of g will be different at different points and cannot be zero at any point.
4. (d) Intensity will be zero inside the spherical shell.
5. (b) The value of g at the height h from the surface of earth

$$g' = g \left(1 - \frac{2h}{R} \right)$$

The value of g at depth x below the surface of earth

$$g' = g \left(1 - \frac{x}{R} \right)$$

These two are given equal, hence $\left(1 - \frac{2h}{R} \right) = \left(1 - \frac{x}{R} \right)$
On solving, we get $x = 2h$

6. (b) We know that $g = \frac{GM}{R^2}$

$$\text{On the planet } g_p = \frac{GM/7}{R^2/4} = \frac{4g}{7} = \frac{4}{7}g$$

$$\text{Hence weight on the planet} = 700 \times \frac{4}{7} = 400 \text{ gm wt}$$

7. (b) For height $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} = 1\%$

$$\text{For depth } \frac{\Delta g}{g} \times 100\% = \frac{d}{R} = \frac{h}{R} = \frac{1}{2}\% = 0.5\%$$

8. (c) $g = \frac{GM}{R^2}$ and $K = \frac{L^2}{2I}$

If mass of the earth and its angular momentum remains, constant then $g \propto \frac{1}{R^2}$ and $K \propto \frac{1}{R^2}$

i.e., if radius of earth decreases by 2%, then g and K both increases by 4%.

9. (a) Radius of earth $R = 6400 \text{ km} \therefore h = \frac{R}{4}$
Acceleration due to gravity at a height h

$$g_h = g \left(\frac{R}{R+h} \right)^2 = g \left(\frac{R}{R+\frac{R}{4}} \right)^2 = \frac{16}{25}g$$

At depth 'd' value of acceleration due to gravity

$$g_d = \frac{1}{2}g_h \quad (\text{According to problem})$$

$$\Rightarrow g_d = \frac{1}{2} \left(\frac{16}{25} \right) g \Rightarrow g \left(1 - \frac{d}{R} \right) = \frac{1}{2} \left(\frac{16}{25} \right) g$$

By solving we get $d = 4.3 \times 10^6 \text{ m}$

10. (d) For scientist A which goes down in a mine

$$g' = g \left(1 - \frac{d}{R} \right)$$

For scientist B, which goes up in the air

$$g' = g \left(1 - \frac{2h}{R} \right)$$

So it is clear that value of g measured by each will decrease at different rates.

11. (b) Gravitational pull depends upon the acceleration due to gravity on that planet.

$$\therefore M_m = \frac{1}{81} M_e, g_m = \frac{1}{6} g_e$$

$$g = \frac{Gm}{R^2} \Rightarrow \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e} \right)^{1/2} = \left(81 \times \frac{1}{6} \right)^{1/2}$$

$$R_e = \frac{9}{\sqrt{6}} R_m$$

12. (c) $g = \frac{GM}{R^2} \therefore g \propto \frac{M}{R^2}$

$$\text{According to problem } M_p = \frac{M_e}{2} \text{ and } R_p = \frac{R_e}{2}$$

$$\therefore \frac{g_p}{g_e} = \left(\frac{M_p}{M_e} \right) \left(\frac{R_e}{R_p} \right)^2 = \left(\frac{1}{2} \right) \times (2)^2 = 2$$

$$\Rightarrow g_p = 2g_e = 2 \times 9.8 = 19.6 \text{ m/s}^2$$

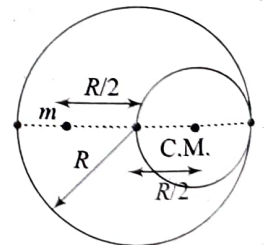
13. (b) Gravitation field at mass 'm' due to full solid sphere

$$\vec{E}_1 = \frac{\rho \vec{r}}{3\epsilon_0} = \frac{\rho R}{6\epsilon_0} \dots \left[\epsilon_0 = \frac{1}{4\pi G} \right]$$

Gravitational field at mass 'm' due to cavity $(-\rho)$

$$\vec{E}_2 = \frac{(-\rho)(R/2)^3}{3\epsilon_0 R^2} \dots \left[\frac{\rho a^3}{3\epsilon_0 r^2} \right]$$

$$= -\frac{(-\rho)R^3}{24\epsilon_0 R^2} = -\frac{\rho R}{24\epsilon_0}$$



Net gravitational field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho R}{6\epsilon_0} - \frac{\rho R}{24\epsilon_0} = \frac{\rho R}{8\epsilon_0}$$

$$\text{Net force on } m \rightarrow F = m\vec{E} = \frac{m\rho R}{8\epsilon_0}$$

$$\text{Here } \rho = \frac{M}{4/3\pi R^3} \text{ and } \epsilon_0 = \frac{1}{4\pi G}$$

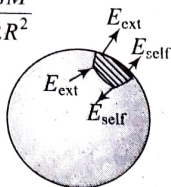
$$\text{Then } F = \frac{3mg}{8}$$

14. (b) Consider a small area (shaded strip)
 here E_{self} = Gravitational field due to this strip
 and E_{ext} = Gravitational field due to the rest of spherical shell.
 E_{in} = Gravitational field just inside the strip
 E_{out} = Gravitational field just outside the strip
 $E_{\text{in}} = E_{\text{ext}} - E_{\text{self}} = 0$
 $E_{\text{ext}} = E_{\text{self}}$

$$E_{\text{out}} = E_{\text{ext}} + E_{\text{self}} = \frac{GM}{R^2} \Rightarrow E_{\text{ext}} = \frac{GM}{2R^2}$$

After the shaded area has been removed
 there is no E_{self} and only E_{ext} .

$$\text{hence, } E_{\text{net}} = E_{\text{ext}} = \frac{GM}{2R^2}$$



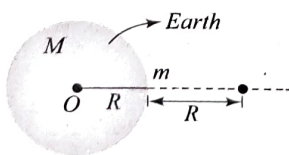
Fill in the Blanks Type

15. $g = \frac{GM}{R^2}$ and $M = \frac{4}{3}\pi R^3 \times \rho$
 $\therefore g = \frac{4}{3}\pi R^3 \times G\rho \Rightarrow \rho = \frac{3g}{4\pi RG}$
16. $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \left(\frac{n-1}{n}\right)R$
17. $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{2} = \frac{R}{R+h}$
 $\Rightarrow R+h = 2R \therefore h = R$
18. $g \propto \frac{1}{R^2}$
 Percentage change in $g = 2$ (percentage change in R)
 $= 2 \times 1.5 = -3\%$
19. $g' = g - \omega^2 R \cos^2 \lambda$
 For weightlessness at equator $\lambda = 0^\circ$ and $g' = 0$
 $\therefore 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800} \text{ rad/sec}$
20. As $g = \frac{GM}{R^2}$, therefore 1% decrease in mass will decrease the value of g by 1%.
 But 1% decrease in radius will increase the value of g by 2%.
 As a whole value of g increase by 1%.

DPP 6.3

Subjective Type

1. Consider the diagram where an object of mass m is raised from the surface of the earth to a distance (height) equal to the radius of the earth (R).



$$\text{Potential energy of the object at the surface of the earth} = -\frac{GMm}{R}$$

$$\text{PE of the object at a height equal to the radius of the earth} = -\frac{GMm}{2R}$$

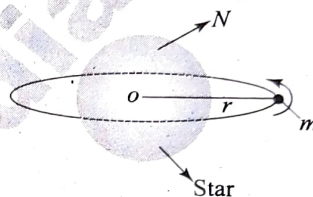
$\therefore$ Gain in PE of the object

$$\begin{aligned} &= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) \\ &= \frac{-GMm + 2GMm}{2R} = +\frac{GMm}{2R} \\ &= \frac{gR^2 \times m}{2R} = \frac{1}{2}mgR \quad (\because GM = gR^2) \end{aligned}$$

2. The situation is shown in the diagram, where a body of mass m is revolving around a star of mass M .

$$\text{Linear velocity of the body } v = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow v \propto \frac{1}{\sqrt{r}}$$



Therefore, when r increases, v decreases.

$$\text{Angular velocity of the body } \omega = \frac{2\pi}{T}$$

According to Kepler's law of period,

$$T^2 \propto r^3 \Rightarrow T = kr^{3/2}$$

where k is a constant

$$\therefore \omega = \frac{2\pi}{kr^{3/2}} \Rightarrow \omega \propto \frac{1}{r^{3/2}}$$

$$\left(\because \omega = \frac{2\pi}{T}\right)$$

Therefore, when r increases, ω decreases.

Kinetic energy of the body

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{GM}{r} = \frac{GMm}{2r}$$

$$\therefore K \propto \frac{1}{r}$$

Therefore, when r increases, KE decreases.

Gravitational potential energy of the body,

$$U = -\frac{GMm}{r} \Rightarrow U \propto -\frac{1}{r}$$

Therefore, when r increases, PE becomes less negative, i.e., increases.

Total energy of the body

$$\begin{aligned} E = \text{KE} + \text{PE} &= \frac{GMm}{2r} \\ &+ \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r} \end{aligned}$$

Therefore, when r increases, total energy becomes less negative, i.e., increases.

S.50 Mechanics II

Angular momentum of the body

$$L = mvr = mr\sqrt{\frac{GM}{r}} = m\sqrt{GM}r$$

$$\therefore L \propto \sqrt{r}$$

Therefore, when r increases, angular momentum L increases.

Single Correct Answer Type

3. (c) Gravitational PE = $m \times$ gravitational potential
 $U = mV$. So the graph of U will be the same as that of V for a spherical shell.
4. (b) If missile is launched with escape velocity, then it will escape from the gravitational field and at infinity its total energy becomes zero.
 But if the velocity of projection is less than escape velocity, then sum of energies will be negative. This shows that attractive force is working on the satellite.

$$5. (d) \Delta U = U_2 - U_1 = \frac{mgh}{1 + \frac{h}{R_e}} = \frac{mgR_e}{1 + \frac{R_e}{R_e}} = \frac{mgR_e}{2}$$

$$\Rightarrow U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$$

$$6. (c) \Delta KE = \Delta U$$

$$\Rightarrow \frac{1}{2}MV^2 = GM_e M \left(\frac{1}{R} - \frac{1}{R+h} \right) \quad (i)$$

$$\text{Also } g = \frac{GM_e}{R^2} \quad (ii)$$

$$\text{On solving (i) and (ii) } h = \frac{R}{\left(\frac{2gR}{V^2} - 1 \right)}$$

7. (b) Let velocities of these masses at r distance from each other be v_1 and v_2 respectively.

By conservation of momentum,

$$\Rightarrow m_1 v_1 - m_2 v_2 = 0 \quad (i)$$

By conservation of energy
 change in PE = change in KE

$$\frac{Gm_1 m_2}{r} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$\Rightarrow \frac{m_1^2 v_1^2}{m_1} + \frac{m_2^2 v_2^2}{m_2} = \frac{2Gm_1 m_2}{r} \quad (ii)$$

On solving equations (i) and (ii),

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1 + m_2)}} \text{ and } v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1 + m_2)}}$$

$$\therefore v_{app} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}} (m_1 + m_2)$$

$$8. (b) \text{ Potential energy } U = \frac{-GMm}{r} = -\frac{GMm}{R+h}$$

$$U_{\text{initial}} = -\frac{GMm}{3R} \text{ and } U_{\text{final}} = -\frac{GMm}{2R}$$

$$\text{Loss in PE} = \text{gain in KE} = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$$

$$9. (c) V = \frac{50}{100} V_e = \frac{1}{2} \sqrt{\frac{2GM}{R}}$$

Apply energy conservation

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mV^2 = -\frac{GMm}{(R+h)}$$

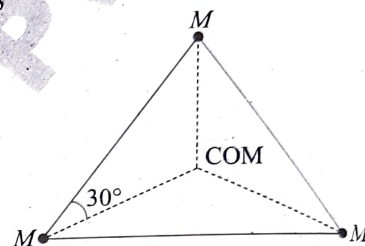
$$v^2 = \frac{2GM}{R} - \frac{2GM}{R+h} \cdot \frac{1}{4} = \frac{2GM}{R} \left(\frac{1}{R} - \frac{1}{R+h} \right)$$

$$\therefore \frac{1}{4R} = \frac{h}{R(R+h)}$$

$$\therefore R+h = 4h \Rightarrow h = \frac{R}{3}$$

10. (b) Stars move around COM. Distance of each star from COM is $\frac{2}{3} \times L \cos 30^\circ = \frac{L}{\sqrt{3}}$

Force on each star M due to the other two



$$2 \cdot \frac{G \cdot M \cdot M}{L^2} \cos 30^\circ = \sqrt{3} \frac{GM^2}{L^2}$$

This acts as centripetal force.

$$\text{Therefore, } \sqrt{3} \frac{GM^2}{L^2} = \frac{mv^2}{(L/\sqrt{3})} \Rightarrow V = \sqrt{\frac{GM}{L}}$$

To dismantle the system, energy equal to total energy of the system must be provided.

$$\text{i.e., } 3 \times \frac{1}{2}MV^2 + \left(3 \times \frac{GM^2}{L} \right) = -\frac{3}{2} \frac{GM^2}{L}$$

Multiple Correct Answers Type

11. (b, c, d)

$$g = \frac{GM}{R^2}, v_e = \sqrt{\frac{2GM}{R}} \text{ and } U = \frac{-GMm}{R}$$

$$\therefore g \propto \frac{M}{R^2}, v_e \propto \sqrt{\frac{M}{R}} \text{ and } U \propto \frac{M}{R}$$

If both mass and radius are increased by 0.5%, then v_0 and U remains unchanged whereas g decrease by 0.5%.

Fill in the Blanks Type

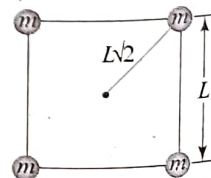
12. (a) Potential at the centre due to single

$$\text{mass} = \frac{-GM}{L/\sqrt{2}}$$

Potential at the centre due to all four masses

$$= -4 \frac{GM}{L/\sqrt{2}} = -4\sqrt{2} \frac{GM}{L}$$

$$= -\sqrt{32} \times \frac{GM}{L}$$



13. Gravitational potential

$$= \int E dx = \int_x^\infty \frac{K}{x^3} dx$$

$$= K \left(\frac{x^{-3+1}}{-3+1} \right)_x^\infty = \left(\frac{-K}{2x^2} \right)_x^\infty = \frac{K}{2x^2}$$

14. $\Delta U = \frac{mgh}{1+h/R}$

Substituting $R = 5h$ we get $\Delta U = \frac{mgh}{1+1/5} = \frac{5}{6} mgh$

15. $v_e = \sqrt{\frac{2GM}{R}} = 100 \Rightarrow \frac{GM}{R} = 5000$

Potential energy $U = -\frac{GMm}{R} = -5000 \text{ J}$

- 16.
- $v_e = \sqrt{2} v_0$
- , i.e. if the orbital velocity of moon is increased by factor of
- $\sqrt{2}$
- , then it will escape out from the gravitational field of earth.

17. $v = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{k_1 \times k_2} = \sqrt{k_1 k_2}$

18. Potential energy of the 1 kg mass which is placed at the earth surface =
- $-\frac{GM}{R}$

Its potential energy at infinite = 0

$\therefore$ Work done = change in potential energy = $\frac{GM}{R}$

DPP 6.4

Single Correct Answer Type

1. (c) $U = \frac{-GMm}{r}$, $K = \frac{GMm}{2r}$ and $E = \frac{-GMm}{2r}$

For a satellite U , K and E vary with r and also U and E remain negative, whereas K remains always positive.

2. (c) Let
- ω_E
- be the angular velocity of earth and
- ω_s
- be the angular velocity of satellite.

Then $6 = \frac{2\pi}{(\omega_E + \omega_s)} \Rightarrow \omega_E + \omega_s = \frac{\pi}{3}$

$\Rightarrow \omega_s = \frac{\pi}{3} - \frac{\pi}{12} \left(\because \omega_E = \frac{2\pi}{24} = \frac{\pi}{12} \text{ rad per hour} \right)$

$\Rightarrow \omega_s = \frac{\pi}{4} \text{ radian per hour}$

3. (c) Given $8 = \frac{2\pi}{\omega_1 + \omega_2} = \frac{2\pi}{\frac{2\pi}{T_1} + \frac{2\pi}{T_2}}$, $T_1 = 24$

hours for earth.

 $\Rightarrow T_2 = 12$ hours (T_2 being the time period of satellite, it will remain same as the distance from the centre of the earth remains constant).

$\Rightarrow T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}} = 24 \text{ hours}$

4. (b) $v = \sqrt{\frac{GM}{r}}$. If $r_1 > r_2$, then $v_1 < v_2$

Orbital speed of satellite does not depend upon the mass of the satellite

5. (b) KE required for satellite to escape from earth's gravitational

field $\frac{1}{2} mv_e^2 = \frac{1}{2} m \left(\sqrt{\frac{2GM}{R}} \right)^2 = \frac{GMm}{R}$

KE required for satellite to move in circular orbit

$\frac{1}{2} mv_0^2 = \frac{1}{2} m \left(\sqrt{\frac{GM}{R}} \right)^2 = \frac{GMm}{2R}$

The ratio between these two energies = 2

6. (b) In the circular motion around the Earth, the centripetal force on the satellite is a gravitational force. Therefore,
- $v^2 = GM/R$
- , where
- M
- is the mass of the Earth,
- R
- is the radius of the orbit of satellite and
- G
- is the universal gravitational constant. Therefore, the kinetic energy increases with the decrease in the radius of the orbit. The gravitational potential energy is negative and decreases with the decrease in radius.

7. (c) $v = \sqrt{\frac{GM}{R+h}}$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$v_2 = \sqrt{\frac{2}{3}} v_1 = \sqrt{\frac{2}{3}} v$

8. (b) $v \propto \frac{1}{\sqrt{r}}$

% increase in speed = $\frac{1}{2}$ (% decrease in radius)

$= \frac{1}{2} (1\%) = 0.5\%$

i.e., speed will increase by 0.5%

9. (a) Since the planet is at the centre, the focus and centre of the elliptical path coincide, and the elliptical path becomes circular and the major axis is nothing but the diameter. For a circular path:

$\frac{mv^2}{r} = \sqrt{\frac{GM}{r^2}} m$

Also $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}$

$\Rightarrow r = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} = \text{Radius}$

$\Rightarrow \text{Diameter (major axis)} = 2 \left(\frac{GMT^2}{4\pi^2} \right)^{1/3}$

10. (d) Orbital radius of satellites
- $r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$U_1 = \frac{-GMm}{r_1}$ and $U_2 = \frac{-GMm}{r_2}$

$K_1 = \frac{GMm}{2r_1}$ and $K_2 = \frac{GMm}{2r_2}$

$$E_1 = \frac{GMm}{2r_1} \text{ and } E_2 = \frac{GMm}{2r_2}$$

$$\therefore \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$$

$$11. (c) \quad (i) \quad T_{st} = 2\pi \sqrt{\frac{(R+h)^3}{GM}} = 2\pi \sqrt{\frac{R}{g}}$$

[As $h \ll R$ and $GM = gR^2$]

$$(ii) \quad T_{ma} = 2\pi \sqrt{\frac{R}{g}}$$

$$(iii) \quad T_{sp} = 2\pi \sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}} = 2\pi \sqrt{\frac{R}{2g}} \quad [\text{As } l = R]$$

$$(iv) \quad T_{is} = 2\pi \sqrt{\frac{R}{g}} \quad [\text{As } l = \infty]$$

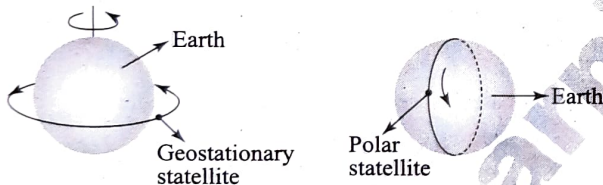
12. (a) Binding energy = -kinetic energy
And if this amount of energy (E_k) given to satellite, then it will escape into outer space

Multiple Correct Answers Type

13. (a, c)

A geostationary satellite is having the same sense of rotation as that of earth, i.e., west-east direction.

A polar satellite goes around the earth's pole in north-south direction.



14. (b, c, d)

$$KE = \frac{GMm}{2R}; PE = -\frac{GMm}{R}$$

$$TE = -\frac{GMm}{2R}$$

$$\text{Angular momentum } mvr = m\sqrt{GMR}$$

Fill in the Blanks Type

15. $T \propto r^{3/2}$. If r becomes double, then time period will become $(2)^{3/2}$ times.

So new time period will be $24 \times 2\sqrt{2}$ hr, i.e., $T = 48\sqrt{2}$

$$16. \quad v = \sqrt{\frac{GM}{R}} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$\therefore \frac{v_A}{v_B} = \frac{3V}{V} = \frac{1}{2} \therefore v_B = 6V$$

17. $v \propto \frac{1}{\sqrt{r}}$. If orbital radius becomes 4 times, then orbital velocity will become half, i.e., $\frac{7}{2} = 3.5$ km/s

18. $v \propto \frac{1}{\sqrt{r}}$. If $r = R$, then $v = V_0$

$$\text{If } r = R + h = R + 3R = 4R \text{ then } v = \frac{V_0}{2} = 0.5V_0$$

19. Given that, $T_1 = 1$ day and $T_2 = 8$ days

$$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{8}{1}\right)^{2/3} = 4$$

$$\Rightarrow r_2 - 4r_1 = 4R$$

20. Orbital velocity $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ and $v_0 = r\omega$

$$\text{This gives } r^3 = \frac{R^2 g}{\omega^2}$$

$$21. \quad T = 2\pi \sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM} (R+h)^3$$

$$\Rightarrow R+h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} \Rightarrow h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} - R$$